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ABSTRACT

This paper analyzes optimal unemployment insurance over the business cycle in a search model in which unemployment stems from matching frictions (in booms) and job rationing (in recessions). Job rationing during recessions introduces two novel effects ignored in previous studies of optimal unemployment insurance. First, job-search efforts have little effect on aggregate unemployment because the number of jobs available is limited, independently of matching frictions. Second, while job-search efforts increase the individual probability of finding a job, they create a negative externality by reducing other jobseekers' probability of finding one of the few available jobs. Both effects are captured by the positive and countercyclical wedge between micro-elasticity and macro-elasticity of unemployment with respect to net rewards from work. We derive a simple optimal unemployment insurance formula expressed in terms of those two elasticities and risk aversion. The formula coincides with the classical Baily-Chetty formula only when unemployment is low, and macro- and micro-elasticity are (almost) equal. The formula implies that the generosity of unemployment insurance should be countercyclical. We illustrate this result by simulating the optimal unemployment insurance over the business cycle in a dynamic stochastic general equilibrium model calibrated with US data.

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1 Introduction

Unemployment insurance (UI) is a key component of social insurance in modern economies, and whether to increase or decrease the generosity of UI during recessions is a critical and controversial public policy question. On the one hand, generous unemployment benefits could discourage job search during recessions and worsen unemployment.[1] On the other hand, high unemployment during recessions does not seem due to a lack of job-search effort but rather a scarcity of jobs.

To characterize optimal unemployment insurance over the business cycle, our paper uses a search-and-matching model in which jobs are endogenously rationed in recessions. We extend the model in Michaillat (2010) to allow for endogenous job-search efforts by unemployed workers. In this model, the combination of real wage rigidity and diminishing marginal returns to labor gives rise to job rationing in an economic equilibrium as well as realistic employment fluctuations over the business cycle. Effectively, unemployment stems from two sources: matching frictions (in booms) and job rationing (in recessions).

Job rationing introduces two novel effects that have been ignored in previous studies of optimal unemployment insurance.[2] The textbook model of optimal UI focuses on the trade-off between insurance value of unemployment benefits and cost of unemployment benefits from reduced job-search effort (Baily 1978; Chetty 2006a). Our first departure from the textbook model is to measure the cost of UI, not solely from lower search efforts, but from higher unemployment that lower search efforts generate in general equilibrium. In our model, the relation between lower search efforts and higher unemployment evolve over the business cycle. In good times, unemployment is due to matching frictions so that higher search effort translates directly into lower unemployment as in the textbook model. In bad times, however, unemployment is due to job rationing while matching frictions contribute little to unemployment, and are not relevant to understanding

[1] For example, the Economist in November 2009 reads: "It may seem heartless to counsel against too much support for the unemployed but incentives matter even when unemployment is high. Firms in rich countries make hires equivalent to some 14-15% of all employment in deep recessions, according to the OECD. More generous benefits will mean vacancies are filled less quickly, pushing up unemployment."

[2] A few recent studies (Andersen and Svarer 2010, 2011; Kiley 2003; Kroft and Notowidigdo 2010; Moyen and Stahler 2009; Sanchez 2008) have started to analyze the issue of optimal UI over the business cycle. We discuss in detail in Section 2 how our model differ from those studies.

unemployment (Michaillat 2010). Accordingly, aggregate job-search efforts have little influence on aggregate unemployment. While unemployment benefits do reduce search efforts in recession, this reduction only increases unemployment negligibly. Our second departure from the textbook model arises from the presence of a negative externality caused by job rationing, which plays a large role in recession. Unemployed workers choose their search effort based on the effect of individual effort on the probability of finding a job, taking the job-finding probability per unit of search effort as given. Yet, since only a limited number of jobs is available, increasing one's probability of finding a job mechanically reduces other jobseekers' probability of finding one of the few available jobs. Thus, individuals tend to search too much for jobs. The government corrects this externality by providing unemployment benefits reducing job-search efforts. Therefore, the cost of UI from higher unemployment (through reduced search effort) decreases in recession, and the value of UI from correcting the job-rationing externality increases in recession. The insurance value of UI from consumption smoothing remains constant over the cycle. Hence, optimal UI is more generous in recessions than in expansions.

We begin the analysis in a one-period, general equilibrium model , whose equilibrium matches the steady-state of the dynamic model introduced later. We can study the equilibrium of this simple static model analytically, and represent it diagrammatically in a labor supply-labor demand framework. We characterize the optimal level of unemployment benefits and tax rates across equilibria parameterized by different levels of technology. Our wage-rigidity assumption implies that when technology is high, wages are relatively low, which drives unemployment down ("an expansion"). Conversely, when technology is low, wages are relatively high, which drives unemployment up ("a recession"). We derive a simple optimal unemployment insurance formula expressed in terms of sufficient statistics that can be empirically estimated: risk version, as well as micro-elasticity and macro-elasticity of unemployment with respect to net reward from work. The micro-elasticity is defined as the elasticity of the probability of unemployment of a single worker whose individual benefits are changed. The macro-elasticity is defined as the elasticity of aggregate unemployment to UI when labor market tightness adjusts. We obtain a formula in terms of these statistics because the macro-elasticity captures the increase in aggregate unemployment caused by UI through lower search effort, while the correction needed for the job-rationing externality is measured by the wedge between micro-elasticity and macro-elasticity. Our formula is very general as it is expressed with sufficient statistics, and is therefore robust to changes in the primitives of the model.[3]

In low-unemployment periods, the macro- and micro-elasticity are (almost) equal, and the formula coincides with the classical Baily-Chetty formula. In high-unemployment periods, the macro-elasticity decreases sharply while the micro-elasticity remains broadly constant. Our formula implies that the generosity of optimal unemployment insurance is countercyclical and higher than in the traditional Baily-Chetty formula for two reasons. First, the elasticity that should be used in the Baily-Chetty formula is the macro-elasticity instead of the micro-elasticity, as only the macro-elasticity of unemployment matters for the government budget. Therefore, during recessions when the macro elasticity is smaller, the optimal replacement rate is higher. Second, the correction for the job-rationing externality depends positively upon the wedge between micro- and macro-elasticity. In recessions, the wedge is large and the optimal replacement rate is even higher. With no concern for insurance (linear utility), the government should still provide UI in recessions.

Next, we use numerical methods to quantify optimal unemployment insurance in a dynamic stochastic environment that accounts fully for rational expectations of firms and workers, as well as the law of motion of unemployment. We calibrate a DSGE model with US data. Technology shocks drive business cycle fluctuations. We simulate the time path of optimal unemployment benefits and labor taxes in response to a technology shock. A 1% decrease in technology requires an increase in the replacement rate of about 1.5%. Thus, the countercyclical pattern of optimal UI is quantitatively large.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a one-period model that transparently illustrates the key economic mechanisms, obtains optimal UI formulas expressed in terms of sufficient statistics, and proposes a numerical illustration. Section 4 uses a DGSE model to obtain more realistic dynamic simulations. Section 5 concludes.

[3]As shown by Chetty (2006a, 2008) in the Baily model, our optimal replacement rate formula expressed in terms of "sufficient statistics" is quite general and carries over to models in which individuals can partially self-insure.

2 Related Literature

Our paper is related to a large literature that analyzes optimal UI theoretically and numerically. Following the work of Baily (1978), a theoretical literature in public economics and macroeconomics has studied optimal UI in search models in which there is a trade-off between insurance and incentives to search.[4]

Papers have analyzed the optimal sequencing of benefits (and taxes to finance them) over time (for example, Hopenhayn and Nicolini 1997; Kocherlakota 2004; Mortensen 1977; Shavell and Weiss 1979; Shimer and Werning 2008). Studies have simulated optimal UI in calibrated models considering various unemployment benefit tools (Fredriksson and Holmlund 2001; Hansen and Imrohoroglu 1992; Lentz 2009; Wang and Williamson 2002). Other papers have characterized optimal UI when unemployment benefits distort wages (Cahuc and Lehmann 2000; Coles and Masters 2006). However, none of those take business cycle fluctuations into account.

Moreover, many papers have considered models with externalities and their consequences for optimal unemployment benefits. General efficiency conditions have been established for search models in Hosios (1990) and Moen (1997). Diamond (1981) shows that, if the distribution of job offerings becomes more attractive when there are more vacancies and more unemployment, then the steady-state equilibrium is not efficient and UI can restore efficiency by making workers more selective in the jobs they accept. Acemoglu (2001) develops a model of noncompetitive labor markets in which good and bad jobs coexist, and in which UI can shift employment toward good jobs and improve efficiency. Marimon and Zilibotti (1999) develop a model in which UI reduces employment but also helps workers to get a suitable job. These three papers assume risk neutrality so UI is just a subsidy for searching longer and improving the quality of job-worker matches. Acemoglu and Shimer (1999) show that, with risk aversion, UI induces workers to seek high-wage jobs with high unemployment risk, and hence improves both risk sharing and output. Spinnewijn (2010) extends the Baily model to the case where unemployed workers have biased beliefs regarding future employment, which calls for corrective unemployment insurance over and above the traditional Baily formula. Kroft (2008) considers a model of optimal UI with endogenous

[4] Fredriksson and Holmlund (2006) offer a recent survey.

take-up driven in part by social interactions that create an externality. He extends the Baily-Chetty formula and shows that the macro-elasticity is the relevant one and that the externality requires an additional correction to the formula. In contrast to these studies, our paper zooms on an externality due to endogenous job rationing that is inherently tied to the business cycle.

A few recent studies have started to study optimal UI over the business cycle. Kiley (2003) and Sanchez (2008) use partial equilibrium models in which benefits are posited to have less distortionary effects in downturns than in booms. In contrast, we construct a model in which such a pattern arises in general equilibrium.

Using general equilibrium models with matching frictions in the labor market, Andersen and Svarer (2010, 2011) and Moyen and Stahler (2009) find countercyclical optimal benefits when the government is not constrained to balance its budget each period, but faces an intertemporal budget constraint instead. In these models, optimal UI is countercyclical because the government uses UI to smooth consumption over the cycle.[5] In contrast, we impose a period-by-period budget balance so that the government cannot use UI as a vehicle for intertemporal consumption smoothing through deficit spending. In spite of this restriction, we find that optimal unemployment benefits are countercyclical.

Kroft and Notowidigdo (2010) propose a model, close in spirit to the traditional Baily model, in which the elasticity of unemployment duration with respect to benefits, and accordingly optimal unemployment benefits, may vary over the business cycle. Since the cyclicality of elasticity is theoretically ambiguous (it depends on the parameters of the model), they propose an empirical estimation. All the variation of optimal UI comes from variation in the micro-elasticity in their Baily formula. In contrast in our model, the micro-elasticity of unemployment with respect to net reward from work is roughly constant; the countercyclicality of optimal UI comes from the procyclicality of the macro-elasticity; this procyclicality arises from the presence of job rationing.

[5]To reinforce this point: Andersen and Svarer (2010) find that optimal benefits should be *procyclical* when they derive comparative statics in the static version of the model (in which there is no room for risk sharing through intertemporal substitution of consumption). In the dynamic model, optimal benefits are countercyclical to allow risk sharing over the business cycle.

3 Static Model

This section presents a one-period model of the labor market and derives a simple optimal unemployment insurance formula that can be expressed in terms of estimable elasticities. The key economic mechanisms are transparent in this model, and its equilibrium can be represented diagrammatically. Furthermore, its equilibrium corresponds to the equilibrium of the dynamic model of Section 4 in which there would be no aggregate shocks and no discounting.

3.1 Description of the economy and equilibrium with UI

3.1.1 Labor market

At the beginning of the period, a fraction $1-U$ of all workers are allocated to a job without having to search. One can think of these $1-U$ workers as incumbent, who were already on the job in the past. A fraction U of all workers have to search for a job. One can think of these U workers as unemployed workers, who did not have a job in the past. Unemployed workers exert an average search effort E per worker. Firms open V vacancies to recruit these jobseekers. The number of matches is given by a constant-returns matching function $m(E \cdot U, V)$ of aggregate effort $E \cdot U$ and vacancies V, differentiable and increasing in both arguments. Conditions on the labor market are summarized by the labor market tightness

$$\theta \equiv \frac{V}{E \cdot U}.$$

The matching technology is such that not all unemployed workers can find a job, and not all vacancies can be filled. An unemployed worker searching with individual effort e finds a job with probability

$$e \cdot f(\theta) \equiv e \cdot \frac{m(E \cdot U, V)}{E \cdot U} = e \cdot m(1, \theta), \tag{1}$$

and a vacancy is filled with probability

$$q(\theta) \equiv \frac{m(E \cdot U, V)}{V} = m(1/\theta, 1) = \frac{f(\theta)}{\theta}.$$

In a tight market, it is easy for jobseekers to find jobs—the job-finding probability per unit of search effort $f(\theta)$ is high—and difficult for firms to hire workers—the job-filling probability $q(\theta)$ is low. We assume that the matching function is Cobb-Douglas, so that

$$f(\theta) = \omega_m \cdot \theta^{1-\eta},\ q(\theta) = \omega_m \cdot \theta^{-\eta},\ \omega_m \in (0, +\infty),\ \eta \in (0,1).$$

3.1.2 Household

The representative household is composed of a mass one of identical workers with utility that depends on consumption C and job search effort E of the form $u(C) - k(E)$ where $u(.)$ is increasing and concave and $k(.)$ is increasing and convex. To simplify derivations, we assume an isoelastic cost of effort

$$k(E) = \omega_k \cdot \frac{E^{1+\kappa}}{1+\kappa},\ \omega_k \in (0, +\infty),\ \kappa \in (0, +\infty).$$

Each individual can neither borrow nor save, and consumes all her income each period.[6] When working, an individual earns wage W. The government taxes earnings at rate t to finance unemployment benefits $b \cdot W$ when unemployed. We denote by $C^e = W \cdot (1-t)$ consumption when employed and by $C^u = b \cdot W$ consumption when unemployed. We denote by $\tau = t + b$ the total implicit tax on work and by $\Delta C = C^e - C^u = (1-\tau) \cdot W$ the net reward from work. τ measures the generosity of the UI system and we refer to τ as the net replacement rate in what follows.[7] Our representative household does not provide insurance to its members, unlike in other standard search-and-matching models (Andolfatto 1996; Merz 1995). Members of the household, however, decide collectively how much to search for jobs. This collective decision imposes that unemployed members take into account the effect of their search effort on their probability *of finding a job* conditional on being unemployed, and on their probability *of being unemployed* in the first place. This theoretical construct aims to capture in a one-period model the fact that in a dynamic model,

[6] We discuss later on how our results extend to the case with endogenous savings or self-insurance paralleling the analysis of Chetty (2006a).

[7] The gross replacement rate is traditionally defined as $b = C^u/W$ while the net replacement rate is defined as $C^u/C^e = b/(1-t) \simeq b + t = \tau$ when the tax rate t is small. As the unemployment rate U is small relative to the working population, t is also small justifying why we call τ the net replacement rate.

higher search effort increases the probability *of finding a job* in the current period, and decreases the probability *of being unemployed* in the future.

More precisely, the household chooses its labor supply N^s to maximize its aggregate utility. Supplying N^s units of labor provides consumption C^e to N^s household members. The $1-N^s$ unemployed household members consume only C^u. Supplying N^s units of labor is costly: while a fraction $1-s$ of the N^s jobs is filled immediately at no cost, a fraction s of the jobs must be filled through matching on the labor market. The fraction s of jobs that are unfilled aims to capture simply the effects of job turnover and matching frictions in our one-period model. A higher s means more job turnover, and hence more job search.[8] The $1-(1-s)\cdot N^s$ household members unemployed at the beginning of the period must exert search effort E to fill $s\cdot N^s$ vacant jobs. Given (1), a fraction $E f(\theta)$ of these jobseekers will find a job. Therefore, the required effort is such that

$$E\cdot f(\theta)\cdot[1-(1-s)N^s] = s\cdot N^s, \tag{2}$$

which imposes a utility cost $k(E)$ on the $1-(1-s)\cdot N^s$ jobseekers.

Equivalently, the household chooses effort E to maximize its aggregate utility

$$-[1-(1-s)\cdot N^s(E,\theta)]\cdot k(E)+[1-N^s(E,\theta)]\cdot u(C^u)+N^s(E,\theta)\cdot u(C^e),$$

where θ, C^u and C^e are taken as given and the labor supply $N^s(E,\theta)$ is given by

$$N^s(E,\theta)=\frac{1}{\frac{s}{E\cdot f(\theta)}+(1-s)}. \tag{3}$$

This labor supply equation comes directly from (2), and determines how search effort E translates into employment for a given labor market tightness θ. $N^s(E,\theta)$ increases with E and θ.

[8]In the dynamic setting of Section 4, s corresponds to the job destruction rate each period. Hence, s is the fraction of employed workers who lose their job each period, and $1-s$ the fraction who retain their job. $1-(1-s)N$ is the number of unemployed workers at the beginning of each period.

Denoting $\Delta u = u(C^e) - u(C^u)$, we can show that the optimal search effort E satisfies

$$k'(E) \cdot \frac{E}{N^s} = \Delta u + (1-s) \cdot k(E), \tag{4}$$

This optimality condition can be rewritten as

$$s\frac{k'(E)}{f(\theta)} + \kappa(1-s)k(E) = \Delta u, \tag{5}$$

which determines optimal effort as a function $E(\theta, \Delta u)$ of the labor market tightness θ and the UI program Δu. $E(\theta, \Delta u)$ increases with θ and Δu.

To summarize, labor supply $N^s(E(\theta, \Delta u), \theta)$ increases with labor market tightness θ and incentive to search Δu. Both properties of the labor supply are illustrated in Figure 1, which plots labor supply curves corresponding to high incentive to search Δu (plain line) and low incentive to search Δu (dotted line) in a price θ-quantity N diagram. As we shall see, in our model with rigid wages, the labor market tightness θ acts as a price to equalize labor supply and labor demand.

3.1.3 Firm

The representative firm produces a consumption good taking price and wage as given.

ASSUMPTION 1 (Diminishing marginal returns to labor)**.** The production function is $F(N, a) = a \cdot N^{\alpha}$, $\alpha \in [0, 1)$. $a > 0$ is the level of technology that proxies for the position in the business cycle.

To capture the effects of job turnover and matching frictions, we assume that while a fraction $1-s$ of the N^d jobs opened by the firm are filled immediately at no cost, the firm must post vacancies to advertise the fraction s of its N^d jobs that are vacant. Keeping a vacancy open has a cost of $r \cdot a$ units of consumption.[9] The recruiting cost $r \in (0, +\infty)$ captures the resources that firms must spend to recruit workers because of matching frictions. We assume away randomness at the firm level: a firm fills a job with certainty by opening $1/q(\theta)$ vacancies, and thus spends $r \cdot a/q(\theta)$

[9] As we shall see, normalizing costs by the technology level a simplifies the derivations.

to fill a job. When the labor market is tighter, a vacancy is less likely to be filled, a firm must post more vacancies to fill a vacant job, and recruiting is more costly.

A firm chooses employment N^d to maximize real profit (the price is normalized to 1)

$$\pi = F(N^d, a) - W \cdot N^d - \frac{r \cdot a}{q(\theta)} \cdot \left(s \cdot N^d\right).$$

The wage W is set once a worker and a firm have matched. Since the marginal product of labor always exceeds the flow value of unemployment, and since the vacancy-posting cost and cost of job-search effort are sunk for firms and workers at the time of matching, there are always mutual gains from trade. There is no compelling theory of wage determination in such an environment (Hall 2005; Shimer 2005). In fact in our one-period model, any wage $\in (0, +\infty)$ could be an equilibrium outcome in a labor market with positive employment. That is, the wage would never result in an inefficient allocation of labor from the joint perspective of the worker-firm pair. This property arises because firms start without any employees and the production function satisfies $\lim_{N \to 0} MPL(N) = +\infty$. Given the indeterminacy of the wage in our frictional labor market, we opt to use the Blanchard and Galí (2010) wage schedule.

ASSUMPTION 2 (Wage rigidity)**.** The wage is $W(a) = w_0 \cdot a^{\gamma}$, $w_0 \in (0, +\infty)$, $\gamma \in [0, 1)$.

The parameter γ captures wage rigidity. If $\gamma = 0$, wages are independent of technology and there is complete wage rigidity. If $\gamma = 1$, wages are proportional to technology and there is no wage rigidity. If $\gamma \in [0, 1)$, when technology is high, wages are relatively low, driving unemployment down as in expansions. Conversely, when technology is low, wages are relatively high, driving unemployment up as in recessions.

From now on, we always denote by F' the marginal product of labor $\partial F / \partial N$. The first-order condition of the firm problem defines implicitly the labor demand $N^d(a, \theta)$ with

$$F'(N^d, a) = W(a) + \frac{s \cdot r \cdot a}{q(\theta)}. \tag{6}$$

Using the functional-form assumptions 1 and 2, and dividing by a, we can rewrite (6) as

$$N^d(\theta, a) = \left\{ \frac{1}{\alpha} \left(w_0 \cdot a^{\gamma-1} + \frac{s \cdot r}{q(\theta)} \right) \right\}^{1/(\alpha-1)}. \tag{7}$$

Since $q(\theta)$ decreases in θ and $F'(N, a)$ decreases in N, the labor demand schedule $N^d(\theta, a)$ decreases with θ when there are diminishing returns to labor ($\alpha < 1$). Moreover, $N^d(\theta, a)$ increases with a when wages are rigid ($\gamma < 1$). Both properties of the labor demand are illustrated in Figure 1, which plots labor demand curves for high (top panel) and low technology (bottom panel) in a price θ-quantity N diagram.

3.1.4 Equilibrium

Given a UI program Δu and technology a, labor market tightness equalizes labor demand to labor supply in equilibrium:

$$N(\Delta u, a) = N^s(E(\theta, \Delta u), \theta) = N^d(\theta, a), \tag{8}$$

where $N(\Delta u, a)$ is equilibrium employment. The equilibrium is illustrated in Figure 1. Equilibrium employment $N(\Delta u, a)$ is given by the intersection of the downward-sloping labor demand curve with the upward-sloping labor supply curve. Labor market tightness acts as a price that equalizes supply and demand in this frictional model. If labor supply is above labor demand, supply and demand can be equalized through a reduction in labor market tightness that both reduces the hiring costs to increase labor demand (equation (7) in which $1/q(\theta)$ increases with θ), and reduces the job-finding probability as well as optimal search effort to reduce labor supply (equations (3) and (5) in which $f(\theta)$ increases with θ).

As showed by Michaillat (2010), job rationing results from the combination of diminishing returns to labor ($\alpha < 1$) and wage rigidity ($\gamma < 1$).[10] In our one-period model, these two assumptions translate into a downward-sloping labor demand curve that shifts down after a negative technology shock as depicted on Figure 1 when moving from the top to bottom panel. As discussed at

[10] Michaillat (2010) defines job rationing as the property of a frictional labor market that does not clear even at the limit when matching frictions disappear.

length in Michaillat (2010), there is ample historical and empirical evidence in favor of these two assumptions. Furthermore, these two assumptions are necessary to provide a realistic description of business cycle fluctuations in the labor market. The rigid-wage assumption ($\gamma < 1$) is critical for labor market tightness θ to depend (positively) on the technology level a, the key ingredient to obtain sufficient unemployment fluctuations in the search model (Hall 2005; Shimer 2005). Our model aims to describe cyclical fluctuations, and the assumption of diminishing returns to labor ($\alpha < 1$) captures the fact that production inputs (especially capital) do not adjust fully to changes in employment at business cycle frequency. If capital and labor are the only production inputs and capital is assumed to be constant in the short run, the production function has diminishing marginal returns to labor as in Assumption 1.

3.2 Optimal unemployment insurance

3.2.1 Government problem

The government chooses the net reward from work $\Delta C = C^e - C^u$ to maximize expected utility

$$N^s(E,\theta) \cdot u(C^u + \Delta C) + [1 - N^s(E,\theta)] \cdot u(C^u) - [1 - (1-s) \cdot N^s(E,\theta)] \cdot k(E) \tag{9}$$

where $N^s(E,\theta)$ is given by labor supply (3), $E(\theta, \Delta u)$ is given by the household's optimal choice of effort (5), θ clears the labor market (8), and the government budget constraint is satisfied. For a given ΔC, the government budget constraint pins down C^u:

$$C^u = N \cdot (W - \Delta C).$$

Note that we assume here that benefits are financed entirely out of wages and that the government cannot tax profits to fund benefits.[11] Using the envelope theorem as E is optimized by the household, and denoting by $\overline{u'} = Nu'(C^e) + (1-N)u'(C^u)$ the average marginal utility, the first order

[11] If profits can be fully taxed, then total wages $N \cdot W$ in equation (3.2.1) should be replaced by the sum of wages and profits which is equal to $F(N,a) - (s \cdot N) \cdot r \cdot a / q(\theta)$. This alternative assumption would generate almost identical results and we consider it the general-equilibrium model of Section 4.

condition for the government choice of ΔC is

$$N \cdot u'(C^e) + \vec{u}' \cdot \frac{dC^u}{d\Delta C} + \frac{\partial N^s}{\partial \theta} \cdot \frac{d\theta}{d\Delta C} \cdot [\Delta u + (1-s) \cdot k(E)] = 0. \quad (10)$$

As we shall see, the first two terms are the classical terms of the Baily-Chetty model. The last term is the correction for the job-rationing externality.

3.2.2 Micro- and macro-elasticity

Introducing elasticities, we can use (10) to express optimal unemployment insurance in terms of estimable parameters. Intuitively, suppose that $d\Delta C > 0$ (unemployment benefits cut). This change create variations in all variables dN, $d\theta$, $d\Delta u$, dC^u, and dE so that all equilibrium conditions continue to be satisfied. The change in effort can be decomposed as $dE = dE_{\Delta u} + dE_\theta$, where $dE_{\Delta u} = (\partial E / \partial \Delta u) d\Delta u$ is a partial-equilibrium change in effort in response to the change in UI, and dE_θ is a general-equilibrium adjustment in effort following the change $d\theta$ in tightness. It is useful to represent labor supply (3) and labor demand (6) in a price θ-quantity N diagram as in Figure 1. Using the labor supply equation (3), we have $dN = dN_E + dN_\theta$ where $dN_E = (\partial N^s / \partial E) dE_{\Delta u}$ and $dN_\theta = (\partial N^s / \partial \theta + (\partial N^s / \partial E)(\partial E / \partial \theta)) d\theta$. $dN_E > 0$ is the increase in aggregate employment due to a positive shift in labor supply, keeping labor market tightness θ constant. The labor supply shifts because the household now exerts more job-search effort, in response to the cut in unemployment benefits. dN_E is represented by the shift A–C in Figure 1. $dN_\theta < 0$ is the reduction in employment that occurs in general equilibrium through a decrease in labor market tightness $d\theta < 0$. dN_θ is represented by the shift C–B in Figure 1. As a combination of these two effects, the general equilibrium increase in employment dN is smaller than the partial equilibrium supply increase in employment dN_E. dN is represented by the shift A–B in Figure 1. The difference between the micro-effect dN_E and the macro-effect dN is dN_θ which arises from job rationing. This decomposition motivates the following definition of the macro and micro elasticities of unemployment $1 - N$ with respect to ΔC.

DEFINITION 1 (Micro-elasticity and macro-elasticity)**.** The *macro-elasticity* of unemployment

$1-N$ with respect to the net reward from work ΔC is defined as:

$$\varepsilon^M = \frac{\Delta C}{1-N} \cdot \frac{dN}{d\Delta C},$$

It measures the percentage increase in unemployment $1-N$ when the net reward from work decreases by 1 percent, assuming all other variables adjust. It is normalized to be positive. The *micro-elasticity* of unemployment with respect to the net reward from work is defined as:

$$\varepsilon^m = \frac{\Delta C}{1-N} \cdot \frac{\partial N^s}{\partial E} \cdot \frac{\partial E}{\partial \Delta u} \cdot \frac{d\Delta u}{d\Delta C}.$$

It measures the percentage increase in unemployment $1-N$ when the net reward from work decreases by 1 percent, ignoring the effect of the general-equilibrium adjustment of θ on N. It is normalized to be positive.

PROPOSITION 1 (Cyclical behavior of micro-elasticity and macro-elasticity)**.**

(i)

$$\varepsilon^m \simeq \frac{u'(C^e) \cdot \Delta C}{\Delta u} \cdot \frac{1}{\kappa+1}, \tag{11}$$

where the approximation is valid for $1-N << 1$ and $s << (1-N)/N$. Hence, for given C^u and C^e, ε^m does not vary systematically with the business cycle (technology level a).

(ii)

$$\varepsilon^m = \varepsilon^M \cdot \left[1 + \frac{1-\eta}{\eta} \cdot (1-\alpha) \cdot U \cdot \frac{1}{1-W/F'} \cdot \left(1 + \frac{U}{\kappa}\right)\right] > \varepsilon^M$$

(iii) *For a given policy $\Delta u = u(C^e) - u(C^u)$, $\varepsilon^m/\varepsilon^M > 1$ varies countercyclically (i.e., decreases with technology a). When a is large (good times), this ratio is close to one. When a is small (bad times), this ratio becomes large.*

The proof is provided in appendix.[12] Three comments should be made. First, our model generates a micro-elasticity of unemployment with respect to net reward from work ε^m that is approximately constant over the business cycle. Thus, the traditional moral-hazard cost of unemployment

[12] As we shall see in our calibration, the assumption $s << (1-N)/N$ is reasonable.

insurance is about constant over the cycle. Second, our model creates a wedge between micro- and macro-elasticity. The macro-elasticity is smaller because of job rationing, which imposes labor market tightness θ and the job-finding probability $f(\theta)$ to adjust downward after a positive shift of the labor supply. Therefore the (general equilibrium) increase in aggregate employment following an increase in aggregate job-search efforts is smaller than the (partial equilibrium) increase in the individual probability to find a job following an increase in individual job-search efforts. Third, the gap between micro and macro-elasticity varies with the business cycle and is small in good times when unemployment is low and largely frictional (as in traditional search models) but large in bad times when unemployment is high and primarily due to job rationing.

Figure 1 illustrates the findings from Proposition 1. The wedge between micro- and macro-elasticity is measured by the distance B–C, which would be positive for any downward-sloping labor demand. The increase in the wedge between micro- and macro-elasticity when technology falls is measured by the increase of the distance B–C between the top panel (high technology) and the bottom panel (low technology). In the bottom panel, employment is bounded at $N = 0.93$ because of job rationing, which makes labor demand intercept the x-axis at $N = 0.93$. Even a large positive shift of labor supply would only have a modest positive effect on aggregate employment.

Results from the empirical literature on the effects of unemployment benefits on unemployment provide support for the three key positive predictions of our theoretical model: (a) positive wedge between micro- and macro-elasticity, (b) acyclical micro-elasticity, (c) countercyclical macro-elasticity. The labor economic literature focuses primarily on the elasticity of unemployment duration with respect to benefits estimated with micro-data (see Krueger and Meyer (2002) for a survey).[13] Although this literature does not distinguish between micro and macro-elasticity, studies comparing individuals with different benefits in the same labor market estimate primarily micro-elasticities while studies comparing individuals with different benefits across labor markets (for example across US states) estimate macro-elasticities.

First, the classical studies by Moffitt (1985) and Meyer (1990) use the same multi-state multi-

[13] A macroeconomic literature uses cross-country and times-series variation to estimate the macro-elasticity of unemployment with respect to benefits. This literature finds a wide range of estimates with no emerging consensus because of both measurement and identification issues (for example, Holmlund 1998; Layard et al. 1991).

year US micro-administrative data but Meyer (1990) includes state fixed effects and hence uses primarily within-state variation in benefits while Moffitt (1985) does not include state fixed effects and hence uses both within- and across-state variation. As a result, Meyer (1990) estimates a micro-elasticity while Moffitt (1985) estimates a mixture of macro- and micro-elasticity. Meyer (1990) finds much higher elasticity estimates (around 0.9) than Moffitt (1985) (around 0.4).[14] This comparison suggests that the micro-elasticity is larger than the macro-elasticity as in our model.

Second, Schmieder et al. (2010) use sharp variation in unemployment benefits duration by age in Germany and a regression discontinuity approach with exhaustive administrative data to identify compellingly the micro-elasticity of duration with respect to benefits. This is the most credible study to date which is able to estimate the micro-elasticity separately for many years. It shows that the micro-elasticity is almost exactly constant over the business cycle in Germany, as in our model.

Third, Moffitt (1985) estimates how the elasticity of duration with respect to benefits varies with the local state unemployment rate and finds that the disincentive effect of UI declines significantly with the unemployment rate in the state. Using survey data, Kroft and Notowidigdo (2010) also find that the elasticity of unemployment durations with respect to benefits is smaller in high-unemployment than in low-unemployment states. As Moffitt (1985) and Kroft and Notowidigdo (2010) use variation in benefits both across and within states, their estimate likely captures a mix of macro- and micro-elasticities. Finally, Arulampalam and Stewart (1995) find much stronger effects of benefits on durations in Britain in 1978 (low unemployment) than in 1987 (high unemployment).[15] Those results therefore suggest that the macro-elasticity may be countercyclical as in our model. We leave the precise estimation of macro- and micro-elasticities over the business cycle, currently lacking from the empirical literature, for future work.

3.2.3 Optimal unemployment insurance formulas

Recall that $\Delta C = (1-\tau)W$ and hence $(W-\Delta C)/\Delta C = \tau/(1-\tau)$.

[14] See Krueger and Meyer (2002), Table 2.5., p. 2349 for a side by side comparison.

[15] Jurajda and Tannery (2003) also find that UI federal expansions in Pennsylvania in the early 1980s have slightly smaller effects on labor supply in a depressed region of the state (Pittsburgh) than in a less depressed region of the state (Philadelphia). The differential response, however, is much smaller than in the studies just mentioned, maybe because there is substantial mobility across those two cities.

PROPOSITION 2 (Optimal UI formulas)**.** *The optimal replacement rate* τ *satisfies*

$$\frac{\tau}{1-\tau} = \frac{N}{\varepsilon^M} \cdot \frac{u'(C^u) - u'(C^e)}{\overline{u'}} + \left(\frac{\varepsilon^m}{\varepsilon^M} - 1\right) \cdot \frac{\kappa \cdot (\kappa+1)}{(\kappa+U)^2} \cdot \left[\frac{\overline{u'} \cdot \Delta C}{\Delta u}\right]^{-1}. \quad (12)$$

With the approximation that $1-N << 1$ *and* $s << (1-N)/N$*, the optimal formula simplifies to*

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^M}\left(\frac{u'(C^u)}{u'(C^e)} - 1\right) + \left(\frac{\varepsilon^m}{\varepsilon^M} - 1\right) \cdot \frac{1}{\frac{u'(C^e)\cdot \Delta C}{\Delta u} - \varepsilon^m}. \quad (13)$$

In both (12) *and* (13)*, the first term on the right-hand-side is the classical Baily-Chetty term while the second term on the right-hand-side is the correction of externality due to job rationing.*

The proof is obtained by re-arranging terms in (10), and is presented in appendix. To illuminate the key economic mechanisms behind the optimal formulas, we present an intuitive derivation. Consider a small increase $d\Delta C$ in the net reward for work—equivalent to a cut in unemployment benefits. The direct mechanical positive welfare effect on workers is $dS_1 = N \cdot u'(C^e) \cdot d\Delta C$ (first term in (10)). But increasing ΔC requires cutting benefits $C^u = N \cdot (W - \Delta C)$ by $dC^u = -N \cdot d\Delta C + (W-\Delta C) \cdot dN = -N \cdot d\Delta C + (1-N) \cdot [(W-\Delta C)/\Delta C] \cdot \varepsilon^M \cdot d\Delta C$, leading to a welfare loss $dS_2 = -N \cdot \overline{u'} \cdot d\Delta C + (1-N) \cdot [(W-\Delta C)/\Delta C] \cdot \varepsilon^M \cdot \overline{u'} \cdot d\Delta C$ (second term in (10)). In the traditional Baily-Chetty model, those are the only two effects, the optimal UI formula is such that $dS_1 + dS_2 = 0$, and there is only the first term in the right hand side of formulas (12) and (13).

However, in our model, there is a third effect due to job loss resulting from the labor tightness adjustment (third term in (10)). Each job lost reduces social welfare by $u(C^e) - s \cdot k(E) - [u(C^u) - k(E)] = \Delta u + (1-s)k(E)$ as each unemployed person incurs search costs $k(E)$ and a fraction s of the employed had to search and incur costs $k(E)$ as well. The individual optimality condition (4), and the isoelastic assumption for $k(E)$ can be used to rewrite the welfare loss per job as $\Delta u + (1-s)k(E) = \Delta u(\kappa+1)/(\kappa+U)$. As discussed above, a small increase $d\Delta C$ leads to a positive shift in labor supply (more search effort), which leads to a reduction in labor market tightness $d\theta$ in general equilibrium because of job rationing. This reduction $d\theta$ destroys dN_θ jobs through two channels: (i) $(\partial N^s/\partial E)(\partial E/\partial \theta)d\theta$ jobs are destroyed through the reduction in search effort—this reduction, however, does not have any welfare effects by the envelope theorem; and (ii) $(\partial N^s/\partial\theta)d\theta$ jobs are destroyed through a reduction in the job-finding probability. By definition

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N} \cdot \left[\frac{\partial N^s}{\partial \theta} + \frac{\partial N^s}{\partial E}\frac{\partial E}{\partial \theta}\right] \cdot \frac{d\theta}{d\Delta C}.$$

But

$$\frac{\partial N^s}{\partial E}\frac{\partial E}{\partial \theta} = \frac{U}{\kappa} \cdot \frac{\partial N^s}{\partial \theta}.$$

Thus, we can show that

$$\frac{\partial N^s}{\partial \theta} d\theta = -d\Delta C \cdot \frac{1-N}{\Delta C} \cdot \frac{\kappa}{\kappa + U} \cdot \left[\varepsilon^m - \varepsilon^M\right].$$

This leads to a welfare loss of $dS_3 = -d\Delta C \cdot (1-N)/\Delta C \cdot [\varepsilon^m - \varepsilon^M] \cdot \Delta u \cdot \kappa(1+\kappa)/(\kappa+U)^2$. This term is negative. It is due to a decrease in job-finding probability (and hence in aggregate employment) when there is more search, which is not internalized by jobseekers. This decrease in job-finding probability is a direct consequence of job rationing.

At the optimum, the sum of the three terms $dS_1 + dS_2 + dS_3$ is zero leading to formula (12). When $1 - N << 1$, then $N \simeq 1$ and hence $\overline{u'} \simeq u'(C^e)$. Furthermore, using the approximation for $\varepsilon^m \simeq (u'(C^e) \cdot \Delta C/\Delta u)/(\kappa+1)$ from Proposition 1, we can obtain formula (13) from formula (12).

Proposition 2 provides a formula for the generosity of unemployment benefits. Four important points should be noted. First, absent any wedge between macro and micro-elasticity, the second term in the right-hand-side of the formulas (12) and (13) vanishes, and we obtain the Baily-Chetty formula. We express the formula in terms of the elasticity of unemployment with respect to the net rewards from work, instead of the elasticity of unemployment with respect to UI benefits C^u because the latter elasticity cannot be constant (it is zero when UI benefits are zero). This allows us also to have a direct formula for the replacement rate τ instead of an implicit formula as in Baily-Chetty.[16] As in Baily-Chetty, the replacement rate decreases with the elasticity (which measures

[16] Our convention is consistent with optimal income tax theory which always expresses optimal tax rates as a function of the elasticity of earnings with respect to one minus the marginal tax rate, instead of the elasticity of earnings with respect to the marginal tax rate. The UI problem of Baily-Chetty is effectively isomorphic to an optimal tax problem with two earnings level (working vs. not working).

the moral hazard effect) and increases with the curvature of the utility function (which measures the value of insurance). If utility is linear, then $u'(C^u) = u'(C^e)$ and there should be no insurance.

Second, in the Baily-Chetty term, the relevant elasticity is the macro elasticity ε^M and not the micro elasticity ε^m that has been conventionally use to calibrate optimal benefits in the public economics literature (Chetty 2008; Gruber 1997). This is because what matters in the trade-off is insurance versus aggregate costs in terms of higher unemployment and hence higher unemployment benefits outlays. Most empirical studies measure the duration of unemployment by comparing unemployed workers in the same economy who face different replacement rates. Therefore, those studies measure the micro-level elasticity of unemployment duration with respect to benefits. Hence, when there is a wedge between the micro and macro elasticity, it is no longer appropriate to use the micro-elasticity estimated from those duration studies.

Third, when there is wedge between micro and macro-elasticity, a second term, directly proportional to the difference between the two elasticities, appears in the optimal UI formula. This term is the correction for the externality imposed by job search in the presence of job rationing. Thus, optimal unemployment insurance is higher than in the Baily-Chetty formula to correct for the negative externality. Even in the absence of any concern for insurance (with linear utility and $u'(C^u) = u'(C^e)$), some unemployment insurance should be provided to correct the externality.

Finally, formula (12) does not depend on functional form assumptions for the utility function, the production function, or the matching function. It is robust to changes in the primitives of the model. The optimal replacement rate can hence be obtained from a few sufficient statistics—micro- and macro-elasticity, curvature of the utility function—that can be empirically estimated. As always, optimal policy formulas can also be used to assess the current UI system. If the current $\tau/(1-\tau)$ is higher than the right-hand-side of formula (12), then increasing the replacement rate is desirable (and conversely).

Propositions 1 and 2 imply that the optimal replacement rate is countercyclical, both through the Baily term and the through the externality term. The Baily term is higher in recessions because the macro-elasticity is smaller. The externality term is higher in recessions because the wedge between micro- and macro-elasticity grows during recessions. Formally, we can state the following proposition (the proof is presented in appendix).

PROPOSITION 3 (Cyclical behavior of optimal replacement rate τ)**.** *Assume log-utility $u(C) = \ln(C)$. Assume that the approximated formulas (11) for ε^m and (13) for τ are valid at the equilibrium (i.e., technology a is high enough such that $1-N << 1$ and $s << (1-N)/N$). Then the optimal net replacement rate τ is countercyclical (i.e., decreases with technology a).*

3.3 Extensions and special cases

3.3.1 Savings and self-insurance

Chetty (2006a,b) shows that the simple Baily formula carries over to models with savings, borrowing constraints, private insurance arrangements, or search and leisure benefits of unemployment. To a large extent, the same generalizations apply to our model and formulas (12) and (13) carry over with minor modifications.

As an illustration, suppose that unemployed workers can increase their consumption with home production. We assume that home production generates additional consumption $h-g(h)$ where $g(h)$ is a convex and increasing function representing costs of home producing h. Let $\tilde{C}^u = C^u + h - g(h)$ be the total consumption of unemployed workers. Individuals choose E and h to maximize

$$-[1-(1-s)N^s(E,\theta)]\,k(E) + (1-N^s(E,\theta))\cdot u(C^u + h - g(h)) + N^s(E,\theta)\cdot u(C^e),$$

and the government chooses the net rewards from work $\Delta C = C^e - C^u$ to maximize expected utility

$$N^s(E,\theta)u(C^u+\Delta C) + (1-N^s(E,\theta))u(C^u+h-g(h)) - [1-(1-s)N^s(E,\theta)]\cdot k(E)$$

where both E and h is chosen optimally by individuals, and subject to the same constraints as in our original problem. Hence, the first order condition for the government problem is exactly identical and formulas (12) and (13) carry over simply by replacing C^u by $\tilde{C}^u$ in each of the utility and marginal utility expressions $u(C^u)$ and $u'(C^u)$.

Although the structure of the formula does not change, the consumption smoothing benefit term

$u'(\tilde{C}^u)/u'(C^e) - 1$ in the first term of formulas (12) and (13) is smaller if individuals can partly self insure, using for example home production. In the extreme case where individuals can fully self-insure and smooth consumption absent a UI program, $u'(\tilde{C}^u)/u'(C^e) = 1$ and there is no reason to have a UI program for insurance purposes. This point was first noted in Baily (1978) and then generalized by Chetty (2006a). It was also used in the calibration of the Baily formula by Gruber (1997) who estimated empirically that each dollar of UI benefits increase consumption by $0.30 when unemployed (instead of dollar for dollar as in our basic model). To keep our numerical illustrations simple, we rule out partial insurance. Thus, our optimal replacement rate is on the high side. We leave more elaborate simulations with partial self-insurance for future work.

3.3.2 Wage responses to UI Benefits

An implicit assumption in our model is that wages are not affected by UI. In particular, wages do not rise if unemployment benefits become more generous. This assumption is supported by empirical evidence (for example, Holmlund 1998; Layard et al. 1991). Nonetheless, it is conceivable that wages respond positively to benefits as more generous benefits strengthen the bargaining power of workers. In the model we have laid out, if we assume that $W(\Delta C)$, an additional term would arise in the first order condition (10) of the government as a change in ΔC affects the government budget constraint through its effect on W. However, this effect is artificial as we have assumed that the government cannot tax profits and affecting wages through benefits in an indirect way to tax profits. If we assume, as we will do in the fully dynamic model of Section 4, that the government can fully tax profits, this effect disappears and the fact that wages depend on W does not affect the optimal formulas (12) and (13). Effectively, W disappears from the government problem when the government controls C^u and C^e and total resources in the economy. The fact that W depends on ΔC, however, affects the macro-elasticity ε^M as changes in wages affect labor demand. Nevertheless, the formulas (12) and (13), expressed in terms of sufficient statistics, remain valid (with a small adjustment to account for the wage change in the government budget constraint). This illustrates the power of the sufficient-statistics approach.

3.3.3 Special cases

To illustrate the economic mechanisms behind our model and situate our work in the existing literature, it is fruitful to consider the three following special cases.

No diminishing returns to labor with $\alpha = 1$: This model was popularized by Hall (2005). While this model can generate large employment fluctuations, it does not exhibit job rationing. With $\alpha = 1$, labor demand (7) implies that labor market tightness θ is independent of employment N. In Figure 1, the labor demand curve would be horizontal. Proposition 1 shows that $\varepsilon^m = \varepsilon^M$, and that these elasticities are broadly constant over the cycle. In that case, the traditional Baily-Chetty formula applies, and Proposition 2 shows that the optimal replacement rate satisfies approximately

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^m}\left(\frac{u'(C^u)}{u'(C^e)} - 1\right). \tag{14}$$

Thus, the optimal replacement rate τ is constant over the business cycle.

A matching function with $\eta = 1$: In that case, $f(\theta) = \omega_m$ is independent of θ which implies that labor market tightness θ does not enter labor supply (3), and does not affect the optimal provision of search effort (5). Hence, there is no job-rationing externality. In Figure 1, the labor supply curve would be vertical. Once more, Proposition 1 shows that $\varepsilon^m = \varepsilon^M$, and that these elasticities are broadly constant over the cycle. The traditional Baily-Chetty formula (14) applies, and the optimal replacement rate τ is constant over the business cycle.

No wage rigidity with $\gamma = 1$: If there is no wage rigidity ($\gamma = 1$), technology a drops out of the labor demand equation (7) and θ and N are independent of a. All labor market variables, and the problem of the government, are therefore independent of technology. While this model generates a wedge between micro- and macro-elasticity, and the externality term is present in the optimal UI formula, the optimal replacement rate is constant over the "business cycle" because this model fails to capture unemployment fluctuations.

3.4 Numerical illustration

In this section, we illustrate our theoretical results numerically. Table 1 summarizes the calibrated parameters. Since we calibrate the parameters in our dynamic model, we defer the presentation of the calibration strategy to Section 4.2, after we have formally introduced the dynamic model. Although these numerical results are obtained in a one-period model abstracting from any dynamics, they are broadly consistent with those obtained in Section 4.3 when we simulate our dynamic stochastic general equilibrium model.

Figure 2 displays in six panels, as a function of technology a (which proxies for the position in the business cycle), (a) the replacement rate $b = C^u/W$, (b) the labor tax $t = 1 - C^e/W$, (c) the net replacement rate (or total implicit tax on work) $\tau = t + b = 1 - \Delta C/W$, (d) the unemployment rate, (e) effort, (f) labor market tightness. Panel (c) confirms that, as our theory predicts, that the net replacement rate is countercyclical, i.e., decreases with a. Quantitatively, the effect is quite significant as the net replacement rate falls from 88% to 65% over the range of technology we consider (which corresponds to variations in the unemployment rate from 11.5% to 3.5% as shown in panel (d)). Panels (a) and (b) show that both components of the net replacement rate—replacement rate b and particularly tax rate t—are countercyclical. Hence, in bad times, it is desirable to increase taxes substantially to finance not only benefits to a larger fraction of the population that is unemployed but also benefits that are more generous per person (relative to prevailing wages). The replacement rate flattens out at almost 80% once unemployment reaches about 10%. Panels (e) and (f) show that both effort and labor market tightness increase sharply with technology a.

Figure 3 displays micro- and macro-elasticities ε^m and ε^M of unemployment with respect to net reward from work as a function of technology a for a constant UI program $\Delta u = \Delta u^*(a = 1)$. It confirms our three theoretical results from Proposition 1. First, the micro elasticity ε^m is close to constant over the business cycle–it varies on a narrow range from 0.33 to 0.39. Second and in contrast, the macro elasticity varies substantially over the business cycle–it varies from 0.04 in very bad times to 0.33 in very good times, an eight-fold increase. Third, macro-elasticity is always smaller than micro-elasticity although the gap is quite small in very good times. Those results carry over (slightly attenuated) if the elasticities are evaluated when the replacement rate is optimal.

Figure 4 displays the optimal net replacement rate τ as a function of technology a that is obtained from three alternative formulas. The first graph is the full optimum from our model (as in Figure 2), the second graph is the replacement rate that is obtained by not including the externality term in our optimum formula (12). As expected, this second replacement rate is lower than the full optimum, and the discrepancy is highest in bad times as the externality term depends on the wedge between micro- and macro-elasticity, which is highest in bad times. The third graph is the replacement rate obtained by not including the externality term and further replacing macro-elasticity by micro-elasticity in the Baily-Chetty term. Note that the replacement rate is almost flat over the business cycle in that case—it varies within a very narrow range from 62% to 64%. This was expected as the micro-elasticity is almost constant over the business cycle. This later case is the standard type of simulations presented in the public economics literature (for example, Gruber 1997). Figure 4 shows that job rationing in recession changes the picture substantially.

Figure 5 further explores this issue and displays the welfare gain (in percent) from using the fully optimal replacement rate vis-a-vis various alternatives as a function of technology. The welfare gains are measured as the percentage-increase in certainty-equivalent consumption C^{eq}, which we define as $U(C^{eq}) \equiv SW$. The welfare gain is plotted relative to the two alternative scenarios analyzed in Figure 4–using the Baily term only with the macro-elasticity, and using the Baily term only with the micro-elasticity. As expected, the welfare gains are minimal in good times when the two elasticities are close and the externality term is hence small. However, the gains are substantial in bad times, especially when using the Baily formula with the micro-elasticity.

Figure 6 compares our main calibration to an alternative calibration with $\alpha = 1$, i.e., a situation with constant returns to scale and no job rationing. This comparison is useful as the influential study of Hall (2005) proposed such a model with $\alpha = 1$. Four points are worth noting. First, panel (a) confirms that, when $\alpha = 1$, micro and macro-elasticity are identical and vary relatively little over the business cycle. Therefore, a powerful test for distinguishing our model from the Hall (2005) model is to assess whether there is a countercyclical and positive gap between the two elasticities. Second, panel (b) confirms that the optimal net replacement rate is almost constant over the business cycle in Hall (2005) while it varies substantially in our model. Third, panel (c) shows that unemployment fluctuates substantially more in Hall (2005) than in our model.[17] Indeed, with constant returns, the fluctuations in unemployment are very large, perhaps even too large relative to plausible technology shocks.[18] Fourth and related, panel (d) shows that the Hall (2005) model also generates much larger variations in labor market tightness than our model that may be implausibly high for plausible technology shocks.

4 Dynamic Model

In this section, we present a dynamic stochastic extension of our one-period model. We calibrate the model using micro- and macro-data for the US labor market. We move beyond the comparative-static results of Proposition 3 by computing impulse response functions of labor market variables and of the optimal unemployment insurance in the fully dynamic model. A byproduct of the quantitative analysis is to verify that the calibrated model describes well the US labor market.

4.1 Description of the economy and equilibrium with UI

The stochastic process for technology $\{a_t\}_{t=0}^{+\infty}$ drives economic fluctuations. The history of technology realizations is $a^t = (a_0, a_1, \ldots, a_t)$.

4.1.1 Labor market flows

At the end of period $t-1$, a fraction s of the N_{t-1} existing worker-job matches are exogenously destroyed. At the beginning of period t, U_t unemployed workers are looking for a job:

$$U_t = 1 - (1-s) \cdot N_{t-1}. \tag{15}$$

[17] Accordingly, we have reduced the range of technology changes in Figure 6.

[18] Hall (2005) study was pathbreaking because it was able to generate sufficient unemployment fluctuations while earlier search models could not. Therefore, generating excessive fluctuations was a virtue in that case.

4.1.2 Individuals

DEFINITION 2 (Individual problem)**.** Given the government policy $\{C_t^e, C_t^u\}_{t=0}^{+\infty}$, and labor market tightness $\{\theta_t\}_{t=0}^{+\infty}$, the individual problem is to choose a collection of stochastic processes $\{E_t, N_t^s\}_{t=0}^{+\infty}$ to maximize the expected utility

$$\mathbb{E}_0\left[\sum_{t=0}^{+\infty}\delta^t\cdot\left(-\left[1-(1-s)N_{t-1}^s\right]\cdot k(E_t)+(1-N_t^s)\cdot u(C_t^u)+N_t^s\cdot u(C_t^e)\right)\right], \tag{16}$$

subject to the law of motion for the probability to be employed in the next period

$$N_t^s=\left[1-(1-s)\cdot N_{t-1}^s\right]\cdot E_t f(\theta_t)+(1-s)\cdot N_{t-1}^s \tag{17}$$

The time t element of household's choice must be measurable with respect to (a^t, N_{-1}).

The optimal effort function therefore satisfies the following Euler equation

$$\left\{\frac{k'(E_t)}{f(\theta_t)}-\delta(1-s)\mathbb{E}\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right]\right\}+\kappa\delta(1-s)\mathbb{E}\left[k(E_{t+1})\right]=\left[u(C_t^e)-u(C_t^u)\right]. \tag{18}$$

4.1.3 Firms

DEFINITION 3 (Firm problem)**.** Given wage, labor market tightness, and technology processes $\{W_t, \theta_t, a_t\}_{t=0}^{+\infty}$, the firm problem is to choose a stochastic process for employment and hiring $\{N_t^d, H_t\}_{t=0}^{+\infty}$ to maximize

$$\mathbb{E}_0\left[\sum_{t=0}^{+\infty}\delta^t\cdot\left(F(N_t^d, a_t)-W_t\cdot N_t^d-\frac{r\cdot a_t}{q(\theta_t)}\cdot H_t\right)\right], \tag{19}$$

The firm faces a constraint on the number of workers employed each period:

$$N_t^d\leq(1-s)\cdot N_{t-1}^d+H_t. \tag{20}$$

The time t element of a firm's choice must be measurable with respect to (a^t, N_{-1}).

We assume that the firm maximization problem is concave. The unique solution to the firm problem is characterized by two equations. First, employment N_t^d and number of hires H_t are related by

$$H_t = N_t^d - (1-s) \cdot N_{t-1}^d \tag{21}$$

because endogenous layoffs never occur in equilibrium. Second, employment N_t^d is determined by the following first-order condition (as in equilibrium $N_t^d < 1$):

$$F'(N_t^d, a_t) = W_t + \frac{r \cdot a_t}{q(\theta_t)} - \delta(1-s)\mathbb{E}_t\left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right] \tag{22}$$

This Euler equation implies that the representative firm hires labor until marginal revenue from hiring equals marginal cost. The marginal revenue is the marginal product of labor F'. The marginal cost is the sum of the wage W_t, the cost of hiring a worker $r \cdot a_t / q(\theta_t)$, minus the discounted cost of hiring next period $\delta \cdot (1-s) \cdot \mathbb{E}_t\left[r \cdot a_{t+1}/q(\theta_{t+1})\right]$.

4.1.4 Equilibrium with unemployment insurance

DEFINITION 4 (Government policy)**.** A government policy is a collection of stochastic processes $\{C_t^e, C_t^u\}_{t=0}^{+\infty}$ that satisfy the government budget constraint for all t and all a^t:

$$F(N_t, a_t) = N_t C_t^e + (1-N_t) C_t^u + \frac{r \cdot a_t}{q(\theta_t)}\left[N_t - (1-s) \cdot N_{t-1}\right]. \tag{23}$$

The t element of the government policy must be measurable with respect to (a^t, N_{-1}).

Importantly, we impose period-by-period budget balance, and hence rule out the possibility for the government to smooth welfare by shifting resources inter-temporally from good times to bad times. This is a natural assumption as we have also ruled out that individuals can save and smooth consumption over time. This also allows us to zoom in on within period insurance-efficiency trade-off.

DEFINITION 5 (Wage process)**.** A wage process is a stochastic process $\{W_t\}_{t=0}^{+\infty}$ defined for all t

and all a^t by

$$W_t = w_0 \cdot a_t^\gamma, \;\; \gamma \in [0,1). \tag{24}$$

DEFINITION 6 (Labor market tightness process)**.** A labor market process is a stochastic process $\{\theta_t\}_{t=0}^{+\infty}$ such that the demand for labor $\{N_t^d\}_{t=0}^{+\infty}$ by firms equals the "supply of labor" $\{N_t^s\}_{t=0}^{+\infty}$ by the household for all t and all a^t

$$N_t = N_t^d = N_t^s. \tag{25}$$

The t element of the labor market tightness must be measurable with respect to (a^t, N_{-1}).

DEFINITION 7 (Decentralized allocation with unemployment insurance)**.** Given initial employment N_{-1} a stochastic process $\{a_t\}_{t=0}^{+\infty}$ for technology, a decentralized allocation with UI program is a collection of stochastic processes $\{E_t, N_t\}_{t=0}^{+\infty}$, a government policy, a wage process, and a labor market tightness process that solve the household and firm problems. Moreover, the wage process satisfies the condition that no worker-employer pair has an unexploited opportunity for mutual improvement. The wage should neither interfere with the formation of an employment match that generates a positive bilateral surplus, nor cause the destruction of such a match .

Therefore, a decentralized allocation with unemployment insurance is a collection of stochastic processes $\{C_t^e, C_t^u, W_t, E_t, N_t, \theta_t\}_{t=0}^{+\infty}$ that satisfies equations (18), (22), (23), (24), (25). We can also derive a sufficient condition for the wage process to always respect the (private) efficiency of all worker-employer matches. This condition would be exactly the same as the one derived by Michaillat (2010): it imposes a lower bound on wage rigidity γ (which depends on α and s) such that inefficient layoffs do not occur with a high enough probability.[19]

4.1.5 Government problem

The unemployment insurance program is history contingent—it is fully contingent on the history of realizations of shocks— and it is taken as given by firms and household. Moreover, we follow

[19] We find that if $\gamma \geq 0.5$, wages are flexible enough to avoid inefficient separations with probability below 1 percent. In other words, inefficient layoffs cannot occur with a negative technology shock of amplitude below 2.3 standard deviations. This sufficient condition is independent from government policy.

Chari et al. (1991) and Aiyagari et al. (2002) and assume that an institutional arrangement exists through which the government can bind itself to the policy plan.

DEFINITION 8 (Government (Ramsey) problem)**.** The government problem is to choose a government policy to maximize

$$\mathbb{E}_0\left[\sum_{t=0}^{+\infty}\delta^t\cdot(-[1-(1-s)N_{t-1}]\cdot k(E_t)+(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)\right], \tag{26}$$

over all decentralized allocations with unemployment insurance. A *Ramsey allocation* is a decentralized allocation that attains the maximum of (26).

The Ramsey allocation is fully described in Proposition A1 in appendix.

4.1.6 Ramsey allocation in the absence of aggregate shocks

We can describe the first-order conditions and constraints of the Ramsey problem in the absence of aggregate shocks. In that case, the Ramsey allocation converge to a constant allocation that is characterized by Proposition 4.

PROPOSITION 4 (Equivalence with one-period model)**.** *The steady state solution of the Ramsey problem in the dynamic model in the absence of aggregate shocks converges to the solution of the Ramsey problem in the one-period model when the discount factor δ converges towards 1. In particular, the optimal approximated formula* (13) *continues to apply in the steady-state of the dynamic model when $1-N<<1$, $s<<1-N$, and $1-\delta<<1$.*

The proof is presented in appendix. This proposition implies that the static model presented Section 3 is the limiting case of the steady-state of the fully dynamic model when there is no discount. This implies that the same economic mechanisms drive the steady-state of the dynamic model. Therefore, in the remaining of this section, we zoom in on the dynamics of the model which could not be analyzed with the static model of Section 3.

4.2 Calibration

We calibrate all parameters at a weekly frequency.[20] Table 1 summarizes the calibrated parameters.

Separation rate: We estimate the job destruction rate from the seasonally-adjusted monthly series for total separation rate in all nonfarm industries constructed by the BLS from the Job Openings and Labor Turnover Survey (JOLTS) for the for the December 2000–June 2010 period.[21] The average separation rate is 0.037, so $s = 0.0093$ at weekly frequency.

Recruiting costs: We estimate the recruiting cost from microdata gathered by Barron et al. (1997) who find that on average, the flow cost of opening a vacancy amounts to 0.098 of a worker's wage. This number accounts only for the labor cost of recruiting. Silva and Toledo (2005) account for other recruiting expenses such as advertising costs, agency fees, and travel costs, to find that 0.42 of a worker's monthly wage is spent on each hire. Unfortunately, they do not report recruiting times. Using the average monthly job-filling rate of 1.3 in JOLTS, 2000–2010, the flow cost of recruiting could be as high as 0.54 of a worker's wage. We calibrate recruiting cost as 0.32 of a worker's wage, the midpoint between the two previous estimates.[22]

Matching function: We picked a Cobb-Douglas matching function. We now set $\eta = 0.7$. Both assumptions are reasonable in light of empirical results surveyed by Petrongolo and Pissarides (2001). To estimate the matching efficiency ω_m, we use steady-state relationships and the normalization normalize $\bar{e} = 1$ to find

$$\omega_m = \frac{s}{1-s} \cdot \frac{1-U}{U} \cdot \theta^{\eta-1}$$

[20] A week is 1/4 of a month and 1/12 of a quarter.

[21] December 2000–June 2010 is the longest period for which JOLTS is available. Comparable data are not available before this date.

[22] Using the average unemployment rate and labor market tightness in JOLTS, we find that 0.89 percent of the total wage bill is spent on recruiting.

We use the seasonally-adjusted, monthly series for the number of vacancies from JOLTS, 2000–2010, and the seasonally-adjusted, monthly unemployment level computed by the BLS from the Current Population Survey (CPS) over the same period, to compute labor market tightness and unemployment. We find $\theta = 0.47$ and $U = 5.9\%$. The resulting estimate of the matching efficiency at weekly frequency is $\omega_m = 0.19$.

Wage rigidity: Next we calibrate the elasticity γ of wages with respect to technology based on estimates obtained from panel data recording wages of individual workers. These microdata are more adequate because they are less prone to composition effects than aggregate data. The survey of the literature by Pissarides (2009) places the productivity-elasticity of wages of existing jobs in the 0.2–0.5 range in US data. A recent study by Haefke et al. (2008) estimates the elasticity of wages of job movers with respect to productivity using panel data for US workers. For a sample of production and supervisory workers over the period 1984–2006, they obtain a productivity-elasticity of total earnings of 0.7. Their estimate, however, is an upper bound on the elasticity of wages as they do not control for the cyclical composition of jobs.[23,24] Therefore, we set $\gamma = 0.5$, a reasonable mid-point in the range of available evidence.

Diminishing marginal returns to labor: So far, we have estimated parameters from microdata or aggregate data, independently of the model. We now calibrate the remaining parameters to match key moments estimated in the data. We calibrate the production function parameter α such that the steady state of the model matches average labor market tightness $\overline{\theta} = 0.47$ and average labor share $\overline{ls} = 0.66$ in US data. We find that $\alpha = 0.67$.[25]

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[23]Workers may accept lower-paid, stop-gap jobs in recessions, and move to better jobs during expansions, biasing the estimated elasticity upwards.

[24]0.7 is an estimate of the elasticity of wages with respect to labor productivity Y/N, whereas γ is the elasticity of wages with respect to technology $a = Y/N^{\alpha}$. While technology and productivity are highly correlated, productivity is less volatile than technology and therefore an estimate of the elasticity of wages with respect to technology would be below 0.7.

[25]We can show that the labor share $\overline{ls} \equiv (\overline{w} \cdot \overline{n})/\overline{y}$ is related to α through the firm's optimality condition by $\overline{ls}\left(s \cdot \frac{0.32}{q(\overline{\theta})} + 1\right) = \alpha$. So α is slightly larger than the labor share because of the recruiting costs.

Wage level: We target a steady-state unemployment rate of $U = 5.9\%$, so we calibrate the wage w_0 to obtain a steady-state employment $\overline{n} = 0.95$, and a steady-state labor share of $\overline{ls} = 0.66$, which imposes $\overline{ls} = w_0 \cdot \overline{n}^{1-\alpha}$. We find $w_0 = 0.67$. Hence, the recruiting cost is $r = 0.32 \cdot w_0 = 0.22$.

Utility function: We choose risk aversion $\sigma = 1$ such that $u(\cdot) = \ln(\cdot)$, which is on the low side of the most compelling estimates Chetty (2006b) but is often used in macro-economic calibration. A lower risk aversion implies a lower value of insurance and hence lower optimal unemployment benefits. Therefore, our risk aversion parameter is conservative.

We choose $\kappa = 1.8$ to match the micro-elasticity of unemployment with respect to benefits estimated in the empirical micro-economic literature. This literature consistently finds large elasticities of duration with respect to benefits levels. For example, the widely cited study by Meyer (1990) estimates an elasticity of 0.9, and this elasticity is used in optimal UI simulations using the Baily formula by Gruber (1997).[26] We normalize the steady-state search effort $\overline{e}$ to 1. For the US, we assume unemployment benefits $b = 60\%$ and labor tax $t = 15\%$, in line with the literature (Chetty 2006b; Gruber 1997).[27] With $\kappa = 1.8$ and $\sigma = 1$, we obtain $\omega_k = 0.49$. With this calibration, we find $\varepsilon^m \simeq 0.36$. The elasticity of unemployment with respect to *benefits* (instead of net reward from work) is $\frac{C^u}{1-N} \frac{\partial(1-N)}{\partial C^u} \simeq 0.9$ in line with Meyer (1990).

There remains considerable uncertainty about some of the parameters and our model abstracts from a number of relevant issues–many of which are explored in the earlier literature. Therefore, this exercise should be seen as an illustration of the magnitudes one could reasonably expect from the rationing theory we have proposed, and how such magnitudes vary with a few key parameters.

[26] This elasticity is conceptually close to a micro-elasticity because it either controls for state unemployment rates or uses state fixed effects.

[27] The UI payroll tax itself is on the order of 3% and hence much smaller than 15% but workers pay a much higher tax rate than unemployed workers because (a) social security taxes do not apply to UI benefits, (b) federal and state income taxes are progressive and workers have substantially higher incomes than the unemployed.

4.3 Numerical solution by log-linearization

To determine the equilibrium of the model for a given UI program, and to solve the Ramsey problem, we log-linearize the model around the steady state with $\overline{a} = 1$. The appendix describes the log-linear model in details. We assume that the log-deviation of technology $\check{a}_t \equiv d\ln(a_t)$ (which represents the percentage-deviation of technology from steady-state) follows an AR(1) process: $\check{a}_{t+1} = \rho \check{a}_t + z_{t+1}$ where z_t is an innovation to technology. We estimate this AR(1) process in US data. We construct log technology as a residual $\log(a) = \log(Y) - \alpha \cdot \log(N)$. Output Y and employment N are seasonally-adjusted quarterly real output and employment in the nonfarm business sector constructed by the Bureau of Labor Statistics (BLS) Major Sector Productivity and Costs (MSPC) program. The sample period is 1964:Q1–2009:Q2. To isolate fluctuations at business cycle frequency , we follow Shimer (2005) and take the difference between log technology and a low frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter 10^5. We estimate detrended log technology as an AR(1) process: $\log(a_{t+1}) = \rho \cdot \log(a_t) + z_{t+1}$ with $z_{t+1} \sim N(0, \nu^2)$. With quarterly data, we obtain an autocorrelation of 0.897 and a conditional standard deviation of 0.0087, which yields $\rho = 0.991$ and $\nu = 0.0026$ at weekly frequency.

4.3.1 Validity of the model

We verify that the model provides a sensible description of reality by comparing important simulated moments to their empirical counterparts. We simulate a model in which the net replacement rate $\tau = 72\%$ is constant over time. This model describes an economy in which the UI program does not respond systematically to the business cycle (tax rate and replacement ratio adjust automatically to ensure budget balance). This net replacement rate allows to keep the same incentives to search $\Delta u = u(C^e) - u(C^u)$ as in the US economy, while having a balanced UI budget. Given the design of our calibration, the steady state of this model matches average US data very well: $\overline{u} = 5.9\%$, $\overline{v/u} = 0.47$, $\overline{e} = 1$.

We focus on second moments of the unemployment rate U, the vacancy/unemployment ratio V/U, real wage W, output Y, and technology a. Table 2 presents empirical moments in US data for the 1964:Q1–2009:Q2 period. Unemployment rate, output, and technology are described above. The real wage is quarterly, average hourly earnings for production and nonsupervisory workers in the nonfarm business sector constructed by the BLS Current Employment Statistics (CES) program, and deflated by the quarterly average of monthly Consumer Price Index (CPI) for all urban households, constructed by BLS. To construct a vacancy series for the 1964–2009 period, we merge the vacancy data for the nonfarm sector from JOLTS for 2001–2010, with the Conference Board help-wanted advertising index for 1964–2001.[28] We take the quarterly average of the monthly vacancy-level series, and divide it by employment to obtain a vacancy-rate series. We construct labor market tightness as the ratio of vacancy to unemployment. All variables are seasonally-adjusted, expressed in logs, and detrended with a HP filter of smoothing parameter 10^5.

Next, we perturb our log-linear model with i.i.d. technology shocks $z_t \sim N(0, 0.0026)$. We obtain weekly series of log-deviations for all the variables. We record values every 12 weeks for quarterly series (Y, W, a). We record values every 4 weeks and take quarterly averages for monthly series (U, U/V). We discard the first 100 weeks of simulation to remove the effect of initial conditions. We keep 50 samples of 182 quarters (2,184 weeks), corresponding to quarterly data from 1964:Q1 to 2009:Q2. Each sample provides estimates of the means of model-generated data. We compute standard deviations of estimated means across samples to assess the precision of model predictions. Table 3 presents the resulting simulated moments. Simulated and empirical moments for technology are similar because we calibrate the technology process to match the data. All other simulated moments are outcomes of the mechanics of the model.

The fit of the model is good along several critical dimensions. First, the model amplifies technology shocks as much as observed in the data because the simulated standard deviation of unemployment (0.126), output (0.024) and of the vacancy-unemployment ratio (0.441) are comparable to the standard deviations estimated in the data (0.168, 0.029, and 0.344, respectively). The response of wages to technology shocks in the model and the data are quite close. A 1-percent decrease in technology decreases wages by 0.7 percent in the data, and 0.5 percent in our model.

[28]The Conference Board index measures the number of help-wanted advertisements in major newspapers. It is a standard proxy for vacancies (for example, Shimer 2005). The merger of both datasets is necessary because JOLTS began only in December 2000 while the Conference Board data become less relevant after 2000, owing to the major role played by the Internet as a source of job advertising.

Third, simulated and empirical slopes of the Beveridge curve are almost identical. The slope, measured by the correlation of unemployment with vacancy, is -0.98 in the model and -0.89 in the data. Last, autocorrelations of all variables in the model match the data. As highlighted by Michaillat (2010), however, labor market variables and wages are too highly correlated with technology.[29]

4.3.2 Impulse response to unexpected and transitory technology shock

We solve the Ramsey problem by log-linearization as well. The log-linear system has three state variables: employment N, as well as the Lagrange multipliers on the household's and firm's optimality conditions. These multipliers impose that the government keep track of the promises made in the previous period to job-searching workers and recruiting firms. The steady state of the Ramsey allocation is $\overline{u} = 6.1\%$, $\overline{v/u} = 0.49$, $\overline{\tau} = 76\%$, $\overline{e} = 0.93$. To confirm the comovements of technology with unemployment insurance in a fully dynamic model, we compute the impulse response functions (IRFs) in the log-linear model.

Figure 7 details the response of policy variables to a negative technology shock of one percent. Both tax rate t and replacement rate b increase slowly after the adverse shock, which drives the increase in the net replacement rate τ. On impact, the net replacement rate increases slightly, and it builds steadily for 80 weeks. At its peak, the net replacement rate τ increases by about 1.3%. The impulse response confirms that the optimal UI replacement rate increases in response to an adverse technology shock. Consumption of employed workers C^e falls on impact, as a consequence of a higher tax rate and lower income per employed worker. C^e then recovers over time towards its steady-state level. Consumption of unemployed workers drops on impact as a consequence of lower income per worker and then rises. C^u becomes higher than its steady-state level after 40 weeks as a consequence of a higher replacement rate. It then remains above its steady-state level until the economy converges back to the steady state. The comparison of the log-deviations of C_t^e and C_t^u implies that the generosity of the UI program increases in recessions since $\Delta C_t = C_t^e - C_t^u$ clearly decreases after an adverse technology shock.

[29]Demand shocks, financial disturbances, and nominal rigidities—absent from the model but empirically important—could explain these discrepancies.

Figure 8 shows the IRFs to a negative technology shock of one percent of labor market variables in the Ramsey allocation, and in an allocation with constant replacement rate $\tau = 72\%$.[30] The behavior of labor market variables is not surprising: unemployment builds slowly and peaks after about 30 weeks. The unemployment-vacancy ratio and tightness $\theta = V/(U \cdot E)$ drop immediately, which reflects the reduction in hiring by firms on impact. Aggregate search effort drops on impact and decreases further over time, in response to both higher benefits and lower labor market tightness. Compared to an economy with constant replacement rate, the increase in replacement rate reduces aggregate search effort. Labor market tightness does not fall as much however. While a higher replacement rate does not increase the amplitude of the peak of unemployment (around week 50), it delays the recovery and imposes higher unemployment than in the economy with constant replacement rate between week 50 and week 250.

Comparing Figure 8 to Figure 2 suggests that our results in the dynamic and static frameworks are broadly consistent. In the dynamic framework, an increase in unemployment from 6% to 7%—that is, a 15% increase from steady state, about 3 times the increase displayed on Figure 8—should be accompanied by an increase in the net replacement rate τ from 76% to 80%—that is, a 4% increase from steady state. This increase is consistent with the slopes of the replacement rate and unemployment schedules on Figure 2.

Next, we compare the dynamic behavior of our baseline model with that of three variants models: a model without job rationing ($\alpha = 1$), a model in which effort and labor market tightness are not linked ($\eta = 1$), and a model with completely flexible wages ($\gamma = 1$). These models are calibrated following the strategy described in Section 4.2. The steady-state Ramsey allocations differ across these models, as described in Table 4. The steady state allocation does not depend on γ, since the wage rigidity γ only affects the dynamics of the model. Thus, the model with $\gamma = 1$ has the same steady-state Ramsey allocation as our baseline model. In a model with $\alpha = 1$, jobs are not rationed. Therefore, an unemployed worker searching for a job does not impose any negative externality (as the number of jobs is not limited, but solely driven by the aggregate search effort). In addition, the macro-elasticity of employment with respect to net rewards from work is higher

[30] We used a model with constant replacement rate $\tau = 72\%$ to assess the validity of our model in Table 3.

than in our baseline model with $\alpha = 0.67$ because the marginal revenue product of labor is independent from employment and as a result, an increase in employment does not trigger a reduction in labor market tightness.[31] As a consequence, it is socially optimal in the model with $\alpha = 1$ to reduce the net replacement rate (to $\overline{\tau} = 56\%$) which increases aggregate search effort (to $\overline{e} = 1.21$) and drives unemployment down (to $\overline{u} = 4.9\%$). In a model with $\eta = 1$, jobs may be rationed but equilibrium employment is directly determined by the labor supply equation (3), without any interaction from the labor-demand side.[32]. Hence, there is no negative search externality, and the macro- and micro-elasticity are equal. In a model with $\eta = 1$, it is therefore socially optimal to have unemployed workers exert large search efforts. As shown on Table 4, it is socially optimal to reduce the net replacement rate (to $\overline{\tau} = 59\%$) which increases aggregate search effort (to $\overline{e} = 1.18$) and drives unemployment down (to $\overline{u} = 5.0\%$).

Figure 9 compares the IRFs across these four models. The dynamics of the Ramsey allocation differ starkly across these four models. We have described the dynamics of our baseline model above. We reproduce them in Figure 9 as a benchmark. In the flexible wage model with $\gamma = 1$, the technology shock has no influence on the Ramsey allocation because wages and recruiting costs are fully flexible. In particular, the net replacement rate τ does not respond to technology shocks. In the model with $\eta = 1$, the UI program does not respond to technology shocks because the policy trade-off is independent from technology (workers' search behavior solely determines employment independently of firms' behavior). Therefore, effort and unemployment (which are solely determined by $u(C^e) - u(C^u)$) do not fluctuate. Only labor market tightness θ responds to the technology shock so that the demand for labor matches the supply of labor ($v/u = e \cdot \theta$ responds automatically). In the model with constant returns $\alpha = 1$, as already explained in Michaillat (2010), the vacancy-unemployment ratio and unemployment respond more strongly to a technology shock than in our model with $\alpha < 1$. The optimal net replacement rate jumps on impact before decreasing rapidly to its steady-state level. On impact, the government reduces the unemployed workers' search effort when firms substitute recruiting inter-temporally from the future to the present in order to smooth recruiting.

[31]On Figure 1, the labor demand curve is horizontal.
[32]On Figure 1, the labor supply curve is vertical.

Finally, we evaluate the sensibility of the results to our calibration. We examine how the the dynamic behavior of the model changes when we modify the calibration of the parameters shaping the utility function (σ, κ) and of the parameters influencing job rationing (α, γ). We first change the calibration of the utility function and study IRFs with less risk aversion ($\sigma = 0.5$), more risk aversion ($\sigma = 2$), a more elastic effort function ($\kappa = 0.9$), and a more inelastic effort function ($\kappa = 3.6$). The steady states differ slightly across these scenarios, as described in Table 4. As shown on Figure 10, the qualitative behavior of the model with these different calibrations remains unchanged. Quantitatively, the net replacement rate increases more after an adverse shock when workers are less risk-averse. The steady-state net replacement rate $\bar{\tau}$, however, is lower. The converse is true when workers are more risk-averse. A change in the elasticity κ of the search cost $k(e)$ has a small effect on the optimal UI. A higher κ slightly reduces the optimal increase in τ.

Next we change the calibration of parameters determining job rationing and study IRFs with more wage rigidity ($\gamma = 0.25$), less wage rigidity ($\gamma = 0.75$), more diminishing marginal returns to labor ($\alpha = 0.5$), and less diminishing marginal returns to labor ($\alpha = 0.84$). The steady states differ slightly across these scenarios as described in Table 4. As shown on Figure 11, the qualitative behavior of the model with these different calibrations remains unchanged. Quantitatively, the net replacement rate increases more after an adverse shock when wages are more rigid or the production function has more diminishing marginal returns to labor . The converse is true when wages are more flexible or marginal returns to labor do not diminish as much with employment. Furthermore, unemployment and vacancy-unemployment ratio respond much more to a technology shock when wages are more rigid (lower γ).

5 Conclusion

This paper analyzes optimal unemployment insurance over the business cycle. We model unemployment as the result from matching frictions (in good times) and job rationing (in bad times). Our model captures the intuitive notion that jobs are scarce during a recession, while retaining the core structure of standard search models. Our central result is that the optimal replacement rate is higher during recessions. We prove this result theoretically, using a simple optimal unemployment insurance formula expressed in terms of micro- and macro-elasticity of unemployment with respect to net reward from work, and risk aversion. Numerical simulations of our model calibrated with US data show that the variation of the optimal replacement rate is quantitatively large over the business cycle.

There are a variety of models with job rationing. Here, we present only one possible source of job rationing: the combination of real wages that only partially adjust to productivity shocks with diminishing marginal returns to labor. We showed that our optimal UI formula can be expressed in terms of sufficient statistics, and that the cyclical behavior of these statistics drove the properties of optimal UI. Since the three fundamental properties of our sufficient statistics—ε^m is acyclical, the wedge $(\varepsilon^m - \varepsilon^M)$ is positive, and the wedge $(\varepsilon^m - \varepsilon^M)$ is countercyclical—are robust to the origin of job rationing, the countercyclicality of the optimal replacement rate is a general property, independent from the specific source of job rationing.

This paper is a first attempt at providing a general-equilibrium framework to study optimal unemployment insurance over the business cycle. Our analysis should be extended in various directions in future work. First and most important, our key economic mechanism hinged crucially on a positive and countercyclical gap between micro- and macro-elasticity. Although there is a large empirical literature on the effects of unemployment insurance on unemployment duration, to our knowledge, no study has estimated separately micro- and macro-elasticities, as well as the gap between the two. This is the most urgent step to test the validity of our normative predictions, and provide most realistic numerical simulations solidly grounded on those estimated elasticities. Conceptually, this test is also important to distinguish between models of unemployment fluctuations without job rationing ($\alpha = 1$ as in Hall (2005)) and models with job rationing ($\alpha < 1$ as in Michaillat (2010), which have very different policy implications.

Second, the model is simplistic in that there are only technology shocks. Future work should explore how other shocks (such as demand shocks or financial disturbances) influence optimal UI. We conjecture that our reduced-form formulas expressed in terms the micro- and macro-elasticities are likely to carry over in a model with other shocks, and a gap between the two elasticities will continue to be a symptom of a job rationing.

Finally, we could extend the analysis to allow to a broader and more realistic set of unemployment insurance tools. In most OECD countries, the government chooses both level and duration of UI. Indeed, in the United States and other countries, the debate about the generosity of UI benefits during recessions focuses primarily on the duration of benefits. Our analysis could be fruitfully extended to a setting in which more generous unemployment insurance implies both higher and longer unemployment benefits, as in Fredriksson and Holmlund (2001).

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A Proofs

A.1 Proof of Proposition 1

By definition, we have:

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N}\left[\frac{\partial N^s}{\partial\theta} + \frac{\partial N^s}{\partial E}\frac{\partial E}{\partial\theta}\right]\frac{d\theta}{d\Delta C}. \tag{A1}$$

The supply equation (3), $N^s(\theta,E) = Ef(\theta)/[s+(1-s)Ef(\theta)]$ implies that $U = 1-(1-s)N = s/[s+(1-s)Ef(\theta)]$, and hence

$$\frac{\partial N^s}{\partial E} = \frac{sf(\theta)}{[s+(1-s)Ef(\theta)]^2} = U\cdot\frac{N}{E}, \tag{A2}$$

$$\frac{\partial N^s}{\partial\theta} = \frac{sEf'(\theta)}{[s+(1-s)Ef(\theta)]^2} = U\cdot(1-\eta)\cdot\frac{N}{\theta}, \tag{A3}$$

where $1-\eta = \theta f'(\theta)/f(\theta)$ is the elasticity of $f(\theta)$ with respect to θ which is constant with a Cobb-Douglas matching function. So we can rewrite (A1) as

$$\varepsilon^M - \varepsilon^m = \frac{\Delta C}{1-N}\cdot U\cdot\frac{N}{\theta}\cdot\left[1-\eta+\frac{\theta}{E}\frac{\partial E}{\partial\theta}\right]\cdot\frac{d\theta}{d\Delta C}. \tag{A4}$$

Using the labor demand equation (7), $F'(N) = W(a)+\frac{s\cdot r\cdot a}{q(\theta)}$, we have $F''\cdot dN = -d\theta\cdot s\cdot r\cdot aq'(\theta)/q(\theta)^2 = (d\theta/\theta)\cdot(F'-W)\cdot\eta$ where $\eta = -\theta q'(\theta)/q(\theta)$ is minus the elasticity of $q(\theta)$. Therefore, $d\theta/dN = [F''/(F'-W)](\theta/\eta) = -[(1-\alpha)/N][F'/(F'-W)](\theta/\eta)$ where $1-\alpha = -NF''/F'$ is minus the elasticity of F' and constant in the Cobb-Douglas case. Hence, we have

$$\frac{N}{\theta}\frac{d\theta}{dN} = -\frac{1-\alpha}{\eta}\frac{F'}{F'-W} \tag{A5}$$

$$\frac{d\theta}{d\Delta C} = \frac{d\theta}{dN}\frac{dN}{d\Delta C} = -\frac{1-\alpha}{\eta}\cdot\frac{F'}{F'-W}\cdot\frac{\theta}{N}\cdot\frac{1-N}{\Delta C}\cdot\varepsilon^M.$$

Finally, the individual first-order condition (5) for E defines implicitly $E(\Delta u,\theta)$ with

$$\frac{\Delta u}{E}\cdot\frac{\partial E}{\partial\Delta u} = \frac{U}{\kappa}+\frac{1-U}{\kappa+1}, \tag{A6}$$

$$\frac{\theta}{E}\cdot\frac{\partial E}{\partial\theta} = \frac{(1-\eta)U}{\kappa}. \tag{A7}$$

Combining those equations, we obtain

$$\varepsilon^M - \varepsilon^m = -\frac{1-\eta}{\eta}\cdot(1-\alpha)\cdot U\cdot\frac{1}{1-W/F'}\cdot\left(1+\frac{U}{\kappa}\right)\cdot\varepsilon^M.$$

This proves item (ii) in Proposition 1. We define

$$R(a,\Delta u) \equiv \varepsilon^m/\varepsilon^M = 1 + \frac{1-\eta}{\eta}\cdot(1-\alpha)\cdot U \cdot \frac{1}{1-W/F'}\cdot\left(1+\frac{U}{\kappa}\right). \tag{A8}$$

We have

$$\frac{dN}{d\Delta C} = \frac{\partial N^s}{\partial E}\frac{dE}{d\Delta C} + \frac{\partial N^s}{\partial\theta}\frac{d\theta}{d\Delta C} = \frac{N\cdot U}{E}\frac{dE}{d\Delta C} - \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\frac{dN}{d\Delta C},$$

and hence,

$$\frac{dN}{d\Delta C} = \frac{N\cdot U/E}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}}\cdot\frac{dE}{d\Delta C},$$
$$\frac{d\theta}{d\Delta C} = -\frac{1-\alpha}{\eta}\frac{F'}{F'-W}\cdot\frac{\theta}{N}\cdot\frac{dN}{d\Delta C} = \frac{-\frac{1-\alpha}{\eta}U\frac{F'}{F'-W}(\theta/E)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}}\cdot\frac{dE}{d\Delta C}.$$

Therefore using (A7) and (A6):

$$\frac{dE}{d\Delta C} = \frac{\partial E}{\partial\Delta u}\frac{d\Delta u}{d\Delta C} + \frac{\partial E}{\partial\theta}\frac{d\theta}{d\Delta C} = \left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)\frac{E}{\Delta u}\frac{d\Delta u}{d\Delta C} - \frac{\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\frac{U}{\kappa}}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}}\cdot\frac{dE}{d\Delta C},$$

which implies

$$\frac{dE}{d\Delta C} = \frac{\left(1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\right)\left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)}\cdot\frac{E}{\Delta u}\cdot\frac{d\Delta u}{d\Delta C},$$
$$\frac{dN}{d\Delta C} = \frac{\frac{U\cdot N}{\Delta u}\left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)}\cdot\frac{d\Delta u}{d\Delta C}.$$

Now, we have

$$\frac{d\Delta u}{d\Delta C} = \frac{d(u'(C^u+\Delta C)-u'(C^u))}{d\Delta C} = u'(C^e) + \Delta u'\frac{dC^u}{d\Delta C}.$$

Using $C^u = N(W-\Delta C)$, this implies

$$\frac{d\Delta u}{d\Delta C} = \vec{u}' + \Delta u'(W-\Delta C)\frac{dN}{d\Delta C} \tag{A9}$$
$$\frac{dN}{d\Delta C} = \frac{\frac{NU}{\Delta u}\left(\frac{U}{\kappa}+\frac{1-U}{\kappa+1}\right)\left(\vec{u}'+\Delta u'(W-\Delta C)\frac{dN}{d\Delta C}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)},$$
$$\frac{dN}{d\Delta C} = \frac{NU\cdot\frac{\vec{u}'}{\Delta u}\left(\frac{U}{\kappa}+\frac{1-U}{1+\kappa}\right)}{1+\frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right)-\frac{\Delta u'\cdot(W-\Delta C)}{\Delta u}\cdot NU\cdot\left(\frac{U}{\kappa}+\frac{1-U}{1+\kappa}\right)}.$$

Therefore,

$$\varepsilon^M = \frac{\frac{NU}{1-N} \cdot \frac{\overline{u'} \cdot \Delta C}{\Delta u}\left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}{1 + \frac{1-\eta}{\eta}(1-\alpha)U\frac{F'}{F'-W}\left(1+\frac{U}{\kappa}\right) - \frac{\Delta u' \cdot (W-\Delta C)}{\Delta u} \cdot NU \cdot \left(\frac{U}{\kappa} + \frac{1-U}{1+\kappa}\right)}$$

$$\varepsilon^M = \frac{\overline{u'} \cdot \Delta C \cdot [U/(1-N)]}{(\kappa+1) \cdot R(a,\Delta u) \cdot \Delta u \cdot \frac{1}{N} \cdot \left[1+\frac{U}{\kappa}\right]^{-1} - \Delta u' \cdot (W - \Delta C) \cdot U} \tag{A10}$$

Finally, using (ii) in Proposition 1, we have,

$$\varepsilon^m = \frac{\overline{u'} \cdot \Delta C \cdot [U/(1-N)]}{(\kappa+1) \cdot \Delta u \cdot \frac{1}{N} \cdot \left[1+\frac{U}{\kappa}\right]^{-1} - \Delta u' \cdot (W - \Delta C) \cdot U \cdot R(a,\Delta u)^{-1}} \tag{A11}$$

Using the approximation, $1-N << 1$ and $s << (1-N)/N$, we have $U = 1-(1-s)N << 1$, $U/\kappa << 1$, $1-N \simeq 1$, and $\overline{u'} \simeq u'(C^e)$. The second term in the factor deliminated by curly brackets in ε^m in (A11) is negligible (relative to the first term). Furthermore, $U/(1-N) = 1 - sN/(1-N) \simeq 1$ as $s << (1-N)/N$, implying $\varepsilon^m \simeq [u'(C^e) \cdot \Delta C/\Delta u]/(\kappa+1)$ and proving (i).

We show that $\partial R/\partial a < 0$ to prove (iii) in the proposition. We first state a lemma describing the response of the equilibrium to a change in technology (comparative statics) for a given UI Δu.

Let $T \equiv F'(N,a)/(F'(N,a) - W(a))$. Using the firm's optimal recruiting behavior (6), we can write

$$T(N,\theta) = \frac{F'(N,a)}{F'(N,a) - W(a)} = \frac{F'(N,a)}{s \cdot r \cdot a} q(\theta(a)) = \frac{\alpha}{s \cdot r} \cdot N^{\alpha-1} \cdot q(\theta).$$

LEMMA A1. *Fix the UI program $\Delta u > 0$. Let $a > 0$. In equilibrium, we have the following comparative-static results: $dN/da > 0$, $dU/da < 0$, $dE/da > 0$, $d\theta/da > 0$, and $dT/da < 0$.*

Proof. For a given UI program Δu, a worker's optimal search behavior (5) implicitly defines search effort as a function $E(\theta)$ such that $\partial E/\partial\theta > 0$. Firm's optimal recruiting behavior (7) implicitly defines labor demand as a function $N^d(a,\theta)$ such that $\partial N^d/\partial a > 0$ and $\partial N^d/\partial\theta < 0$. Equation (3) defines labor supply as a function $N^s(E(\theta),\theta)$ such that $\partial N^s/\partial E > 0$ and $\partial N^s/\partial\theta > 0$—that is, $dN^s/d\theta > 0$. The equilibrium condition $N^s(\theta) = N^d(a,\theta)$ implicitly defines labor market tightness as a function $\theta(a)$. Differentiating this condition with respect to a yields

$$\frac{dN^s}{d\theta}\frac{d\theta}{da} = \frac{\partial N^d}{\partial a} + \frac{\partial N^d}{\partial\theta}\frac{d\theta}{da}$$

$$\frac{d\theta}{da} = \frac{\partial N^d}{\partial a} \cdot \left[\frac{dN^s}{d\theta} - \frac{\partial N^d}{\partial\theta}\right]^{-1}.$$

Thus $d\theta/da > 0$. In equilibrium, $N(a) = N^s(\theta(a))$ so $dN/da > 0$ and $dU/da = -(1-s)(dN/da) < 0$. Since $E(a) = E(\theta(a))$, $dE/da > 0$. Since $\partial T/\partial\theta < 0$ and $\partial T/\partial N < 0$, $dT/da < 0$. □

Using Lemma A1, we can immediately conclude that $\partial R/\partial a < 0$.

A.2 Proof of Proposition 2

First, using $C^u = N(W - \Delta C)$,

$$\frac{dC^u}{d\Delta C} = (1-N)\frac{\tau}{1-\tau}\cdot \varepsilon^M - N.$$

Second, using the optimality condition (4), and the isoelastic assumption for $k(E)$, we can write

$$\Delta u + (1-s)\cdot k(E) = \Delta u \cdot \frac{\kappa+1}{\kappa+U}.$$

Lastly, the combination of (A1), (A3), and (A7) yields

$$\frac{\partial N^s}{\partial \theta}\cdot\frac{d\theta}{d\Delta C} = \left(\varepsilon^M - \varepsilon^m\right)\frac{1-N}{\Delta C}\frac{\kappa}{\kappa+U}.$$

Reshuffling these terms in (10) and dividing the equation by $(1-N)\varepsilon^M \overline{u}'$ yields (12).

A.3 Proof of Proposition 3

Consider optimality condition (12). It can be written as

$$Q(\tau) = Z(a,\tau) \tag{A12}$$

with $a \in (0,+\infty)$ and $\tau \in [0,1]$. For any a, we assume that (A12) admits a unique solution $\tau^*(a)$. Equivalently, we assume that $Q(\tau)$ and $Z(a,\tau)$ cross only once for $\tau \in [0,1]$.

LEMMA A2. $\lim_{\tau\to 1} Q(\tau) = +\infty$ *and for any* $a > 0$, $\lim_{\tau\to 1} Z(a,\tau) = M < +\infty$

Proof. We consider two cases.

First case: $C^e/C^u \to K > 1$ Then $\Delta u = \ln(C^e/C^u) \to \ln(K) > 0$. In that case all variables are $\in (0,+\infty)$. Moreover, $\Delta C, \Delta u, \Delta u'$ are bounded away from zero. Accordingly, the elasticities ε^m and $\varepsilon^M \in (0,+\infty)$. Then $\lim_{\tau\to 1} Z(a,\tau) \in (0,+\infty)$.

Second case: $C^e/C^u \to 1$ Then $\Delta u = \ln(C^e/C^u) \to 0$, which complicates the analysis. We need to prove that $Q(a,\tau)$ converges to a finite limit. Since $\Delta u \to 0$, $U \to 1$, $E \to 0$, $N \to 0$, $\theta \to +\infty$. Hence $R(a,\Delta u) \to R \equiv 1 + (1-\eta)/\eta(1-\alpha)(\kappa+1)/\kappa$. Budget constraint imposes $(1-N)bW + N(1-t)W = NW$, or $t = b(1-N)/N$. Since $\tau = t+b$, $\tau = b/N$, so that $C^u = \tau NW$ and $C^e = [1-(1-N)\tau]W$. When $\tau \to 1$, $C^u \sim NW$ and $C^e \sim NW$. We have $U/(1-N) \to 1$, $(\kappa+1)(1+U/\kappa)^{-1} \to \kappa$, $N(W-\Delta C) \sim NW$, $\overline{u}' \sim u'(C^u) = 1/C^u$, $\overline{u}'\Delta C \sim \Delta C/C^u$, $\Delta u = \ln(C^e/C^u) \sim C^e/C^u - 1 = \Delta C/C^u$, $-\Delta u' = \Delta C/(C^e\cdot C^u)$ so that $-\Delta u'\cdot NW \sim \Delta C/C^u$. Accordingly, $\varepsilon^M/N \sim 1/(\kappa R+1)$. Moreover, $-\Delta u'/\overline{u}' \to 0$ when $\tau \to 1$, $(\varepsilon^m/\varepsilon^M - 1)\kappa(\kappa+1)/(\kappa+U)^2 \to (1-\eta)/\eta(1-\alpha)$, and $\overline{u}'\Delta C/\Delta u \sim 1$. Hence, $\lim_{\tau\to 1} Z(a,\tau) \in (0,+\infty)$. $\square$

LEMMA A3. *Let $a > 0$ and let $\tau^*(a)$ be the unique solution to (A12). For all $\tau < \tau^*(a)$, $Q(\tau) < Z(a,\tau)$ and for all $\tau > \tau^*(a)$, $Q(\tau) > Z(a,\tau)$.*

Proof. Using the results from Lemma A2 and the single-crossing assumption. □

As the government budget is $b(1-N)W = tNW$, $1-N << 1$ implies that $t << 1$ and hence $C^u/C^e = b/(1-t) \simeq b+t = \tau$. Therefore, $\Delta u = \ln(C^e/C^u) = -\ln(\tau)$. We denote again $R(a,\tau) = \varepsilon^m/\varepsilon^M$. Using the approximation(11) for ε^m from Proposition 1, we can write the micro-elasticity as a function of τ:

$$\varepsilon^m(\tau) \simeq -\frac{1}{\kappa+1}\frac{1-\tau}{\ln(\tau)}.$$

Therefore, the approximated optimal formula (13) can be rewritten as:

$$\frac{\tau}{1-\tau} \simeq \frac{1}{\varepsilon^m(\tau)}\left\{R(a,\tau)\cdot\frac{1-\tau}{\tau}+\frac{1}{\kappa}\left(R(a,\tau)-1\right)\right\}.$$

We write the equilibrium condition as $Q(\tau) = \hat{Z}(a,\tau)$. From Proposition 1, we know that $\partial R(a,\tau)/\partial a < 0$ for all $\tau \in [0,1]$. We can use the result from Proposition 1 because the partial derivative wrt a taking Δu as given is the same as the partial derivative wrt a taking τ as given, since Δu depends *only on τ and not on a*. Therefore $\partial\hat{Z}/\partial a < 0$ for all τ.

Consider a decrease in technology from a to $a' < a$. $Q(\tau^*(a)) = \hat{Z}(a,\tau^*(a)) < \hat{Z}(a',\tau^*(a))$. Lemma A3 (which applies to $\hat{Z}$ if a' close enough to a, when our approximations are valid) implies that $\tau^*(a) < \tau^*(a')$. Thus, $\partial\tau^*/\partial a < 0$.

B Derivation of the Ramsey Allocation in the Dynamic Model

B.1 Firm and household problem

The unconditional probability of observing an history a^t is given by the probability measure $\mu_t(a^t)$.

Representative firm: Endogenous layoffs never occur in equilibrium so the Lagrangian of the firm problem is

$$\mathcal{L} = \sum_{t=0}^{+\infty}\delta^t\sum_{a^t}\mu_t(a^t)\cdot\left\{F(N_t^d,a_t) - W_t\cdot N_t^d - \frac{r\cdot a_t}{q(\theta_t)}\cdot\left[N_t^d-(1-s)\cdot N_{t-1}^d\right]\right\}. \tag{A13}$$

I assume that the firm maximization problem is concave and admits an interior solution (which will always be the case in equilibrium). Immediately, we can show that employment N_t^d is determined by first-order condition (22).

Representative household: The Lagrangian of the household's problem is

$$\sum_{t=0}^{+\infty}\delta^t\sum_{a^t}\mu_t(a^t)\cdot\Big\{-\left[1-(1-s)N_{t-1}^s\right]\cdot k(E_t)+(1-N_t^s)\cdot u(C_t^u)+N_t^s\cdot u(C_t^e)$$
$$+A_t\left\{\left[1-(1-s)\cdot N_{t-1}^s\right]\cdot E_t f(\theta_t)+(1-s)\cdot N_{t-1}^s-N_t^s\right\}\Big\},$$

where $N_t^s(a^t)$ is the probability to be employed in period t after period a^t and $\{A_t(a^t)\}$ is a collection of Lagrange multipliers. The first-order condition with respect to effort in the current period e_t gives:

$$k'(E_t)=f(\theta_t)\cdot A_t.$$

The first-order condition with respect to expected employment status N_t^s yields

$$A_t=[u(C_t^e)-u(C_t^u)]+\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]+\delta\cdot(1-s)\cdot\mathbb{E}_t\left[A_{t+1}\left(1-E_{t+1}f(\theta_{t+1})\right)\right]$$
$$\frac{k'(E_t)}{f(\theta_t)}=[u(C_t^e)-u(C_t^u)]+\delta\cdot(1-s)\cdot\mathbb{E}_t\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right]-\delta\cdot(1-s)(\kappa+1)\cdot\mathbb{E}_t\left[k(E_{t+1})\right]+\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]$$

Thus, the optimal effort function therefore satisfies the Euler equation (18).

B.2 Ramsey Problem

The maximization of the government is over a collection of sequences $\{N_t(a^t),E_t(a^t),\theta_t(a^t),C_t^e(a^t),C_t^u(a^t),\ \forall a^t\}_{t=0}^{+\infty}$. We can form a Lagrangian:

$$\sum_{t=0}^{+\infty}\delta^t\sum_{a^t}\mu_t(a^t)\cdot\Big\{\left(-(1-(1-s)N_{t-1})\cdot k(E_t)+(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)\right.$$
$$+A_t\left[F(N_t,a_t)-N_tC_t^e-(1-N_t)C_t^u-\frac{r\cdot a_t}{q(\theta_t)}\left[N_t-(1-s)\cdot N_{t-1}\right]\right]$$
$$+B_t\left[[u(C_t^e)-u(C_t^u)]-\frac{k'(E_t)}{f(\theta_t)}+\delta(1-s)\mathbb{E}_t\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right]-\kappa\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]\right]$$
$$+C_t\left[F'(N_t,a_t)-W_t-\frac{r\cdot a_t}{q(\theta_t)}+\delta(1-s)\mathbb{E}_t\left[\frac{r\cdot a_{t+1}}{q(\theta_{t+1})}\right]\right]$$
$$+D_t\left[(1-(1-s)\cdot N_{t-1})\cdot E_t f(\theta_t)+(1-s)\cdot N_{t-1}-N_t\right]\Big\}$$

where $\{A_t(a^t),B_t(a^t),C_t(a^t),D_t(a^t),\ \forall a^t\}_{t=0}^{+\infty}$ are sequences of Lagrange multipliers, and

$$\mathbb{E}_t\left[X_{t+1}\right]=\sum_{a^{t+1}|a^t}\frac{\mu_{t+1}(a^{t+1})}{\mu_t(a^t)}X_{t+1}(a^{t+1})$$

is conditional expectation operator. We rewrite the Lagrangian as:

$$\begin{aligned}
&\sum_{t=0}^{+\infty}\delta^t\sum_{a^t}\mu_t(a^t)\cdot\Bigg\{(-(1-(1-s)N_{t-1})\cdot k(E_t)+(1-N_t)\cdot u(C_t^u)+N_t\cdot u(C_t^e)\\
&+A_t\left[F(N_t,a_t)-N_tC_t^e-(1-N_t)C_t^u-\frac{r\cdot a_t}{q(\theta_t)}\left[N_t-(1-s)\cdot N_{t-1}\right]\right]\\
&+B_t\left[\left[u(C_t^e)-u(C_t^u)\right]-\frac{k'(E_t)}{f(\theta_t)}\right]+B_{t-1}(1-s)\left[\frac{k'(E_t)}{f(\theta_t)}-\kappa k(E_t)\right]\\
&+C_t\left[F'(N_t,a_t)-W_t-\frac{r\cdot a_t}{q(\theta_t)}\right]+C_{t-1}(1-s)\left[\frac{r\cdot a_t}{q(\theta_t)}\right]\\
&+D_t\left[(1-(1-s)\cdot N_{t-1})\cdot E_tf(\theta_t)+(1-s)\cdot N_{t-1}-N_t\right]\Bigg\}
\end{aligned}$$

First order conditions of the Ramsey problem with respect to $N_t(a^t)$ for $t\geq 0$:

$$\begin{aligned}
0&=u(C_t^e)-u(C_t^u)+\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]\\
&-D_t+(1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1})\right]\\
&+C_t\cdot F''(N_t,a_t)\\
&+A_t\left\{F'(N_t,a_t)-(C_t^e-C_t^u)-\frac{ra_t}{q(\theta_t)}\right\}+(1-s)\delta\mathbb{E}_t\left[A_{t+1}\cdot\frac{ra_{t+1}}{q(\theta_{t+1})}\right]\\
D_t&=u(C_t^e)-u(C_t^u)+\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right]+(1-s)\mathbb{E}_t\left[D_{t+1}\cdot(1-E_{t+1}f(\theta_{t+1}))\right]\\
&+C_t\cdot F''(N_t,a_t)+A_t\left\{W(a_t)-(C_t^e-C_t^u)\right\}+(1-s)\delta\mathbb{E}_t\left[(A_{t+1}-A_t)\cdot\frac{ra_{t+1}}{q(\theta_{t+1})}\right]
\end{aligned}$$

With respect to C_t^e for $t\geq 0$:

$$\begin{aligned}
0&=N_t\cdot u'(C_t^e)+B_t\cdot u'(C_t^e)-A_t\cdot N_t\\
A_t&=u'(C_t^e)\left(1+\frac{B_t}{N_t}\right)
\end{aligned}$$

With respect to C_t^u for $t\geq 0$:

$$\begin{aligned}
0&=(1-N_t)\cdot u'(C_t^u)-B_t\cdot u'(C_t^u)-A_t\cdot(1-N_t)\\
A_t&=u'(C_t^u)\left(1-\frac{B_t}{1-N_t}\right)
\end{aligned}$$

With respect to E_t for $t \geq 0$:

$$0 = -U_t \cdot k'(E_t) - B_t \frac{k''(E_t)}{f(\theta_t)} + (1-s)B_{t-1}\frac{k''(E_t)}{f(\theta_t)} - \kappa(1-s)B_{t-1}k'(E_t) + D_t \cdot U_t \cdot f(\theta_t)$$

$$0 = -U_t \cdot k'(E_t) + \frac{k''(E_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(1-s)B_{t-1}k'(E_t) + D_t \cdot U_t \cdot f(\theta_t)$$

$$0 = -(\kappa+1)U_t \cdot k(E_t) + \kappa\frac{k'(E_t)}{f(\theta_t)}\left((1-s)B_{t-1} - B_t\right) - \kappa(\kappa+1)(1-s)B_{t-1}k(E_t) + D_t \cdot \cdot E_t \cdot U_t \cdot f(\theta_t)$$

$$\frac{D_t \cdot H_t}{(\kappa+1)k(E_t)} = U_t + \kappa\frac{1}{E_t f(\theta_t)}\left[B_t - (1-s)B_{t-1}\right] + \kappa(1-s)B_{t-1}$$

With respect to θ_t for $t \geq 0$:

$$\begin{aligned}
0 = &-A_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} \cdot H_t \\
&+(1-\eta)B_t\frac{k'(E_t)}{\theta_t \cdot f(\theta_t)} - (1-\eta)(1-s) \cdot B_{t-1}\frac{k'(E_t)}{\theta_t \cdot f(\theta_t)} \\
&-C_t \cdot \eta \cdot \frac{r \cdot a_t}{f(\theta_t)} + C_{t-1} \cdot (1-s) \cdot \eta\frac{r \cdot a_t}{f(\theta_t)} \\
&+D_t \cdot U_t \cdot (1-\eta) \cdot E_t q(\theta_t)
\end{aligned}$$

$$\begin{aligned}
0 = &-A_t \cdot \frac{r \cdot a_t}{q(\theta_t)} \cdot H_t \\
&+\frac{1-\eta}{\eta}\frac{k'(E_t)}{f(\theta_t)}\left(B_t - (1-s) \cdot B_{t-1}\right) \\
&-\frac{r \cdot a_t}{q(\theta_t)}\left(C_t - (1-s) \cdot C_{t-1}\right) \\
&+D_t U_t \cdot \frac{1-\eta}{\eta} \cdot E_t f(\theta_t)
\end{aligned}$$

$$0 = H_t \cdot \left(-A_t \cdot \frac{r \cdot a_t}{q(\theta_t)} + D_t\frac{1-\eta}{\eta}\right) + \frac{1-\eta}{\eta}\frac{k'(E_t)}{f(\theta_t)}\left(B_t - (1-s) \cdot B_{t-1}\right) - \frac{r \cdot a_t}{q(\theta_t)}\left(C_t - (1-s) \cdot C_{t-1}\right)$$

$$0 = H_t \cdot \left(-A_t \cdot r \cdot a_t + D_t q(\theta_t)\frac{1-\eta}{\eta}\right) + \frac{1-\eta}{\eta}\frac{k'(E_t)}{\theta_t}\left(B_t - (1-s) \cdot B_{t-1}\right) - r \cdot a_t\left(C_t - (1-s) \cdot C_{t-1}\right)$$

The following proposition summarizes the results.

PROPOSITION A1 (Characterization of Ramsey allocation)**.** *The Ramsey allocation* $\{C_t^e, C_t^u, \theta_t, N_t, E_t\}_{t=0}^{+\infty}$ *and the sequences of Lagrange multipliers from the government problem* $\{A_t, B_t, C_t, D_t\}_{t=0}^{+\infty}$ *are* *characterized by the following constraints:*

$$0 = F(N_t, a_t) - N_t C_t^e - (1-N_t)C_t^u - \frac{r \cdot a_t}{q(\theta_t)} H_t \tag{A14}$$

$$0 = [u(C_t^e) - u(C_t^u)] - \frac{k'(E_t)}{f(\theta_t)} + \delta(1-s)\mathbb{E}_t\left[\frac{k'(E_{t+1})}{f(\theta_{t+1})}\right] - \kappa\delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right] \tag{A15}$$

$$0 = F'(N_t, a_t) - W(a_t) - \frac{r \cdot a_t}{q(\theta_t)} + \delta(1-s)\mathbb{E}_t\left[\frac{r \cdot a_{t+1}}{q(\theta_{t+1})}\right] \tag{A16}$$

$$0 = U_t \cdot E_t f(\theta_t) - H_t, \tag{A17}$$

and the following first-order conditions with respect to N_t, C_t^e, C_t^u, E_t, θ_t *(respectively):*

$$\begin{aligned} D_t = {} & u(C_t^e) - u(C_t^u) + \delta(1-s)\mathbb{E}_t\left[k(E_{t+1})\right] + (1-s)\mathbb{E}_t\left[D_{t+1} \cdot (1 - E_{t+1} f(\theta_{t+1})\right] \\ & + C_t \cdot F''(N_t, a_t) + A_t\left\{W(a_t) - (C_t^e - C_t^u)\right\} + (1-s)\delta\mathbb{E}_t\left[(A_{t+1} - A_t) \cdot \frac{ra_{t+1}}{q(\theta_{t+1})}\right] \end{aligned} \tag{A18}$$

$$A_t = u'(C_t^e)\left(1 + \frac{B_t}{N_t}\right)$$

$$A_t = u'(C_t^u)\left(1 - \frac{B_t}{1-N_t}\right)$$

$$\frac{D_t \cdot H_t}{(\kappa+1)k(E_t)} = U_t + \kappa\frac{1}{E_t f(\theta_t)}\left[B_t - (1-s)B_{t-1}\right] + \kappa(1-s)B_{t-1} \tag{A19}$$

$$0 = H_t \cdot \left(-A_t \cdot r \cdot a_t + D_t q(\theta_t)\frac{1-\eta}{\eta}\right) + \frac{1-\eta}{\eta}\frac{k'(E_t)}{\theta_t}\left(B_t - (1-s) \cdot B_{t-1}\right) - r \cdot a_t\left(C_t - (1-s) \cdot C_{t-1}\right) \tag{A20}$$

Equivalently:

$$A_t = \left\{\frac{N_t}{u'(C_t^e)} + \frac{1-N_t}{u'(C_t^e)}\right\}^{-1} \tag{A21}$$

$$B_t = N_t \cdot (1-N_t)\left(\frac{1}{u'(C_t^e)} - \frac{1}{u'(C_t^u)}\right)A_t \tag{A22}$$

COROLLARY A1 (Ramsey allocation in the absence of aggregate shocks)**.** *The Ramsey allocation in the absence of aggregate shocks is constant:* $\{C^e, C^u, N, \theta, E, A, B, C, D\}$ *is characterized by* *the following equations:*

$$[1-\delta(1-s)]\frac{k'(E)}{f(\theta)}+\delta(1-s)\kappa\cdot k(E)=[u(C^e)-u(C^u)]$$

$$N=\frac{1}{(1-s)+s/\cdot E\cdot f(\theta)}$$

$$NC^e+(1-N)C^u=F(N,a)-\frac{s\cdot r\cdot a}{q(\theta)}\cdot N$$

$$0=F'(N,a)-W(a)-[1-\delta\cdot(1-s)]\frac{r\cdot a}{q(\theta)}$$

$$D(1-(1-s)\cdot(1-E\cdot f(\theta)))=(u(C^e)-u(C^u))+\delta(1-s)k(E)+C\cdot F''(N,a)+A\{W(a)-(C^e-C^u)\}$$

$$D=\frac{k'(E)}{f(\theta)}\left\{1+\frac{B}{N}\cdot\frac{\kappa}{U}\right\}$$

$$C/N=\frac{1-\eta}{\eta}\frac{k'(E)}{r\cdot a\cdot\theta}\left(1+\frac{B}{N}\cdot\left(\frac{\kappa}{U}+1\right)\right)-A$$

$$A=\left\{\frac{N}{u'(C^e)}+\frac{1-N}{u'(C^e)}\right\}^{-1}$$

$$B=N\cdot(1-N)\left(u'(C^u)-u'(C^e)\right)\frac{A}{u'(C^e)\cdot u'(C^u)}$$

Proof. The first-order condition with respect to E becomes (when the labor market is in steady state, $Ef(\theta)U=H$):

$$\frac{D\cdot U\cdot f(\theta)}{k'(E)}=U+\kappa(1-s)B+\kappa\frac{s}{Ef(\theta)}B$$

$$D=\frac{k'(E)}{f(\theta)}\left\{1+\kappa(1-s)\frac{B}{U}+\kappa\cdot\frac{B}{N}\right\}$$

$$D=\frac{k'(E)}{f(\theta)}\left\{1+\frac{B}{N}\cdot\frac{\kappa}{U}\right\}$$

The first-order condition with respect to θ becomes

$$0=s\cdot N\cdot\left(-A\cdot\frac{r\cdot a}{q(\theta)}+D\frac{1-\eta}{\eta}\right)+\frac{1-\eta}{\eta}\frac{k'(E)}{f(\theta)}s\cdot B-\frac{r\cdot a}{q(\theta)}\cdot s\cdot C$$

$$\frac{r\cdot a}{q(\theta)}\cdot(A\cdot N+C)=\frac{1-\eta}{\eta}\left(N\cdot D+\frac{k'(E)}{f(\theta)}\cdot B\right)$$

$$\frac{r\cdot a}{q(\theta)}\cdot(A\cdot N+C)=\frac{1-\eta}{\eta}\frac{k'(E)}{f(\theta)}\left(N+B\cdot\left(\frac{\kappa}{U}+1\right)\right)$$

$$C=\frac{1-\eta}{\eta}\frac{k'(E)}{r\cdot a\cdot\theta}\left(N+B\cdot\left(\frac{\kappa}{U}+1\right)\right)-A\cdot N$$

$\square$

COROLLARY A2 (Equivalence with one-period model). *The Ramsey allocation in the dynamic model in the absence of aggregate shocks converges to the solution of the Ramsey problem in the one-period model when the discount factor* δ *converges towards* 1.

Proof. The incentive-compatibility constraint in the one-period model is given by (4). Notice that, using $E\,f(\theta)U = s\cdot N$,

$$\begin{aligned}
k'(E)\frac{E}{N} &= \frac{k'(E)}{f(\theta)}\frac{E\,f(\theta)U}{U\cdot N} = s\frac{k'(E)}{f(\theta)}\frac{1}{U} \\
-(1-s)k(E) &= \kappa(1-s)k(E) - (1+\kappa)(1-s)k(E) \\
&= \kappa(1-s)k(E) - (1-s)\frac{k'(E)}{f(\theta)}\frac{E\,f(\theta)U}{U} \\
&= \kappa(1-s)k(E) - s\frac{k'(E)}{f(\theta)}\frac{(1-s)N}{U} \\
k'(E)\frac{E}{N} - (1-s)k(E) &= s\frac{k'(E)}{f(\theta)}\left[\frac{1}{U} - \frac{(1-s)N}{U}\right] + \kappa(1-s)k(E) = s\frac{k'(E)}{f(\theta)} + \kappa(1-s)k(E)
\end{aligned}$$

So the incentive-compatibility constraint in the one-period model can be rewritten as (5). We can form a Lagrangian:

$$\begin{aligned}
\mathcal{L} = &-(1-(1-s)N)\cdot k(E) + (1-N)\cdot u(C^u) + N\cdot u(C^e) \\
&+ A\left[F(N,a) - NC^e - (1-N)C^u - \frac{r\cdot a}{q(\theta)}s\cdot N\right] \\
&+ B\left[[u(C^e) - u(C^u)] - s\frac{k'(e)}{f(\theta)} - \kappa(1-s)k(e)\right] \\
&+ C\left[F'(N,a) - W(a) - \frac{s\cdot r\cdot a}{q(\theta)}\right] \\
&+ D\left[(1-(1-s)\cdot N)\cdot E\,f(\theta) - s\cdot N\right]
\end{aligned}$$

By inspection, it appears that the allocation solving the system of equations described in corollary A1 for $\delta = 1$ also solves constraints and first-order conditions associated with the maximization of the Lagrangian in the one-period model.If both optimization problems are convex, then they admit the same unique solution. $\square$

B.3 Impulse response to unexpected, transitory, technology shock

We first characterize the steady state of the model, and then describe the log-linearized equilibrium conditions around this steady state. $\bar{x}$ denotes the steady-state value of variable X_t. The steady-state Ramsey allocation $\left\{\overline{c_e},\overline{c_u},\overline{n},\overline{\theta},\overline{A},\overline{B},\overline{C},\overline{D}\right\}$ is characterized by Corollary A1 when $a = \overline{a} = 1$.

Moreover $\overline{h} = s\overline{n}$ and $\overline{u} = 1-(1-s)\overline{n}$. $\check{x}_t \equiv d\log(X_t)$ denotes the logarithmic deviation of variable X_t. The equilibrium is described by the following system of log-linearized equations:

- Definition of labor market tightness:

$$\check{u}_t + \check{e}_t + (1-\eta)\cdot\check{\theta}_t - \check{h}_t = 0$$

- Definition of unemployment:

$$\check{u}_t + \zeta\cdot\check{n}_{t-1} = 0$$

  where $\zeta = \frac{1-\overline{u}}{\overline{u}}$.

- Law of motion of employment:

$$(1-s)\cdot\check{n}_{t-1} + s\cdot\check{h}_t - \check{n}_t = 0$$

- Resource constraint:

$$\check{a}_t + \alpha\check{n}_t - \left\{q_1\cdot\left(\check{h}_t + \eta\cdot\check{\theta}_t + \check{a}_t\right) + q_2\cdot\left\{p_1\left(\check{n}_t + c\check{e}_t\right) + p_2\left(-\nu\check{n}_t + c\check{u}_t\right)\right\}\right\} = 0,$$

  with $q_1 = \frac{r}{q(\overline{\theta})}\cdot s\cdot\overline{n}^{1-\alpha}$, $p_1 = \frac{\overline{nc_e}}{(1-\overline{n})\overline{c_u}+\overline{nc_e}}$, $\nu = \frac{\overline{n}}{1-\overline{n}}$, $q_2 = 1-q_1$, and $p_2 = 1-p_1$.

- Firm's Euler equation:

$$-\check{a}_t + (1-\alpha)\cdot\check{n}_t + r_1\cdot\gamma\cdot\check{a}_t + r_2\cdot\left(\eta\cdot\check{\theta}_t + \check{a}_t\right) + r_3\mathbb{E}_t\left[\eta\cdot\check{\theta}_{t+1} + \check{a}_{t+1}\right] = 0$$

  with $r_1 = w_0\cdot\frac{1}{\alpha\cdot\overline{a}}\cdot\overline{n}^{1-\alpha}$, $r_2 = \frac{c}{q(\overline{\theta})}\cdot\frac{1}{\alpha}\cdot\overline{n}^{1-\alpha}$, and $r_3 = 1-r_1-r_2$.

- Productivity shock:

$$\check{a}_t = \rho\cdot\check{a}_{t-1} + z_t$$

- Household's Euler equation:

$$\varepsilon_e s_1 c\check{e}_t + \varepsilon_u s_2 c\check{u}_t - \left\{t_2\left[\frac{1}{1-\delta(1-s)}\left[\kappa\check{e}_t - (1-\eta)\check{\theta}_t\right] - \frac{\delta(1-s)}{1-\delta(1-s)}\mathbb{E}\left[\kappa\check{e}_{t+1} - (1-\eta)\check{\theta}_{t+1}\right]\right] \right.$$
$$\left. + t_1(1+\kappa)\mathbb{E}\left[\check{e}_{t+1}\right]\right\} = 0$$

  where we define the elasticity of $u(\cdot)$ around steady-state

$$\varepsilon_i = \left.\frac{d\ln(u(x))}{d\ln(x)}\right|_{x=\overline{c_i}}$$

  and $s_1 = u(\overline{ce})/\overline{\Delta u}$, $s_2 = 1-s_1$, $t_2 = 1-t_1$, and $t_1 = \frac{\kappa\delta(1-s)k(\overline{e})}{\overline{\Delta u}}$.

- Lagrangian A_t:

$$\check{A}_t + u_1\left(\check{n}_t - \varepsilon'_e \check{ce}_t\right) + u_2\left(-\nu\check{n}_t - \varepsilon'_u \check{cu}_t\right) = 0$$

where we define the elasticity of $u(\cdot)$ around steady-state

$$\varepsilon'_i = \left.\frac{d\ln(u'(x))}{d\ln(x)}\right|_{x=\overline{c_i}}$$

and where $u_1 = \frac{\overline{n}/u'(\overline{c_e})}{\overline{n}/u'(\overline{c_e})+(1-\overline{n})/u'(\overline{c_u})}$, and $u_2 = 1-u_1$.

- Lagrangian B_t:

$$\check{B}_t - \left[(1-\nu)\check{n}_t + \check{A}_t - \left(\varepsilon'_e \check{ce}_t\right) - \left(\varepsilon'_u \check{cu}_t\right) + \left\{\varepsilon'_e v_1 \check{ce}_t + \varepsilon'_u v_2 \check{cu}_t\right\}\right] = 0$$

where $v_1 = \frac{u'(\overline{c_e})}{u'(\overline{c_e})-u'(\overline{c_u})}$, and $v_2 = 1-v_1$.

- Lagrangian D_t defined by equation (A19):

$$\check{D}_t + \check{u}_t + (1-\eta)\check{\theta}_t - \kappa\check{e}_t - \left[w_2\check{u}_t + w_3\check{B}_{t-1} - w_4\left[(1-\eta)\check{\theta}_t + \check{e}_t - \left\{\frac{1}{s}\check{B}_t - \frac{1-s}{s}\check{B}_{t-1}\right\}\right]\right] = 0$$

where $w_1 = \frac{\overline{u}\cdot\overline{D}\cdot f(\overline{\theta})}{k'(\overline{e})}$, and $w_2 = \overline{u}/w_1$, $w_3 = \kappa\cdot(1-s)\cdot\overline{B}/w_1$, $w_4 = 1-w_2-w_3$.

- Lagrangian C_t defined by equation (A20):

$$\check{h}_t + x_4\left(\check{A}_t + \check{a}_t\right) + x_5\left(-\eta\check{\theta}_t + \check{D}_t\right) - x_6\left[-\check{\theta}_t + \kappa\check{e}_t + \frac{1}{s}\check{B}_t - \frac{1-s}{s}\check{B}_{t-1}\right] - x_7\left[\check{a}_t + \frac{1}{s}\check{C}_t - \frac{1-s}{s}\check{C}_{t-1}\right] = 0$$

where $x_1 = \overline{A}r - q(\overline{\theta})\cdot\overline{D}\cdot\frac{1-\eta}{\eta}$, $x_2 = \frac{1-\eta}{\eta}\cdot s\cdot\overline{B}\cdot\frac{k'(\overline{e})}{\overline{\theta}}$, $x_3 = \cdot s\cdot r\cdot\overline{C}$, and $x_4 = \overline{A}\cdot r/x_1$, $x_5 = 1-x_4$, $x_6 = x_2/(x_1\overline{h})$, $x_7 = 1-x_6$.

- Optimality condition from first-order condition with respect to N_t

$$\check{D}_t - \Big\{y_1\left(\varepsilon_e z_1 \check{ce}_t + \varepsilon_u z_2 \check{cu}_t\right) + y_2(1+\kappa)\mathbb{E}\left[\check{e}_{t+1}\right] + y_3\mathbb{E}\left[\check{D}_{t+1} - z_6\left(\check{e}_{t+1} + (1-\eta)\check{\theta}_{t+1}\right)\right]$$
$$+ y_4\left(\check{C}_t + \check{a}_t + (\alpha-2)\check{n}_t\right) + y_5\left(\check{A}_t + \left\{z_3\gamma\check{a}_t + z_4\check{ce}_t + z_5\check{cu}_t\right\}\right)\Big\} = 0$$

where ε_i is defined as above and $z_1 = \frac{u(\overline{c_e})}{u(\overline{c_e})-u(\overline{c_u})}$, $y_1 = \frac{u(\overline{c_e})-u(\overline{c_u})}{\overline{D}}$, $y_2 = \delta(1-s)\frac{k(\overline{e})}{\overline{D}}$, $y_3 = (1-s)\left(1-\overline{e}f(\overline{\theta})\right)$, $z_3 = \frac{w_0}{w_0-(\overline{c_e}-\overline{c_u})}$, $z_4 = -\frac{\overline{c_e}}{w_0-(\overline{c_e}-\overline{c_u})}$, $y_4 = -\alpha(1-\alpha)\frac{\overline{C}\overline{n}^{\alpha-2}}{\overline{D}}$, $z_6 = \frac{\overline{e}f(\overline{\theta})}{1-\overline{e}f(\overline{\theta})}$, and $z_2 = 1-z_1$, $z_5 = 1-z_3-z_4$, $y_5 = 1-y_1-y_2-y_3-y_4$.

Once we have solved the log-linear system, we can recover the log-deviations of the policy variables. Let P_t be the consumption per employed worker

$$\begin{aligned} P_t &= \frac{1}{N_t} \cdot \left(Y_t - \frac{r \cdot a_t}{q(\theta_t)} H_t \right) \\ t_t &= 1 - \frac{C_t^e}{P_t} \\ b_t &= \frac{C_t^u}{P_t} \\ \tau_t &= t_t + b_t \end{aligned}$$

These relations gives the steady-state values $\overline{p}, \overline{b}, \overline{t}, \overline{\tau}$. Then we infer

$$\begin{aligned} \check{p}_t &= -\check{n}_t + \left(a_1 \check{y}_t + a_2 \left(\check{a}_t + \eta \check{\theta}_t \right) + \check{h}_t \right) \right) \\ \check{t}_t &= -b_1 \left(\check{c}_{et} - \check{p}_t \right) \\ \check{b}_t &= \check{c}_{ut} - \check{p}_t \\ \check{\tau}_t &= c_1 \check{t}_t + c_2 \check{b}_t \end{aligned} \tag{A23}$$

where $a_1 = \overline{y}/(\overline{n}\overline{p})$, $a_2 = 1 - a_1$, $b_1 = (\overline{c_e}/\overline{p})/\overline{t}$, $c_1 = \overline{t}/\overline{\tau}$, $c_2 = 1 = c_1$. We can also determine the log-deviation of the certainty equivalent consumption defined by $u(C_t) \equiv SW_t$. Then

$$\frac{\overline{sw}}{\overline{c}u'(\overline{c})} \check{sw}_t = \check{c}_t.$$

B.3.1 Log-linear model under constant UI program

In that case τ is constant, and the government does not pick the UI program optimally. In the log-linear system, we eliminate the 4 Lagrange multipliers $\check{A}_t, \check{D}_t, \check{C}_t, \check{D}_t$ and 4 log-linear equations that give these multipliers. We also replace the equation giving the optimal UI program by an equation that ensures that τ remain constant:

$$\check{\tau}_t = 0,$$

where $\check{\tau}_t$ is a linear function of the log-deviations in the system, as described by (A23).

C Tables and Graphs

Table 1: PARAMETER VALUES IN SIMULATIONS.

	Interpretation	Value	Source
$\overline{a}$	Steady-state technology	1	Normalization
$\overline{e}$	Steady-state effort	1	Normalization
s	Separation rate	0.95%	JOLTS, 2000–2010
δ	Discount factor	0.999	Corresponds to 5% annually
ω_m	Efficiency of matching	0.19	JOLTS, 2000–2010
η	Elasticity of job-filling	0.7	Petrongolo and Pissarides (2001)
γ	Real wage rigidity	0.5	Pissarides (2009), Haefke et al. (2008)
c	Recruiting costs	0.21	$0.32\times$ steady-state wage
w_0	Steady-state real wage	0.67	Matches steady-state unemployment of 5.9%
α	Returns to labor	0.67	Matches labor share of 0.66
σ	Risk aversion	1	Chetty (2006b)
κ	Search elasticity	1.8	Meyer (1990)
ω_k	Searching cost	0.87	Matches $\overline{e}=1$ for $t=15\%$ and $b=60\%$

Table 2: SUMMARY STATISTICS, QUARTERLY US DATA, 1964–2009.

	U	V/U	W	Y	a
Standard Deviation	0.168	0.344	0.021	0.029	0.019
Autocorrelation	0.914	0.923	0.950	0.892	0.871
Correlation	1	-0.968	-0.239	-0.826	-0.478
	–	1	0.220	0.828	0.479
	–	–	1	0.512	0.646
	–	–	–	1	0.831
	–	–	–	–	1

Notes: All data are seasonally adjusted. The sample period is 1964:Q1–2009:Q2. Unemployment rate U is quarterly average of monthly series constructed by the BLS from the CPS. Vacancy rate V is quarterly average of monthly series constructed by merging data constructed by the BLS from the JOLTS and data from the Conference Board, as detailed in the text. Vacancy-unemployment ratio V/U is the ratio of vacancy to unemployment. Real wage W is quarterly, average hourly earnings of production and non-supervisory workers in the private sector, constructed by the BLS CES program, and deflated by the quarterly average of monthly CPI for all urban households, constructed by BLS. Y is quarterly real output in the nonfarm business sector constructed by the BLS MSPC program. $\log(a)$ is computed as the residual $\log(Y)-\alpha\cdot\log(N)$ where N is quarterly employment in the nonfarm business sector constructed by the BLS MSPC program. All variables are reported in log as deviations from an HP trend with smoothing parameter 10^5.

Table 3: SIMULATED MOMENTS WITH TECHNOLOGY SHOCKS AND CONSTANT UI PROGRAM

	U	V/U	W	Y	a
Standard deviation	0.126	0.441	0.009	0.024	0.018
	(0.023)	(0.076)	(0.002)	(0.004)	(0.003)
Autocorrelation	0.936	0.909	0.877	0.894	0.877
	(0.023)	(0.031)	(0.040)	(0.035)	(0.040)
	1	-0.977	-0.985	-0.991	-0.985
	–	1	0.974	0.972	0.974
Correlation	–	–	1	0.999	1.000
	–	–	–	1	0.999
	–	–	–	–	1

Notes: Results from simulating the log-linearized model under constant UI program such that $\tau = 72\%$ with stochastic technology. All variables are reported as logarithmic deviations from steady state. Simulated standard errors (standard deviations across 50 simulations) are reported in parentheses.

Table 4: STEADY-STATE RAMSEY ALLOCATION ACROSS CALIBRATIONS

Calibration	Parameter values			Steady state			
	ω_m	ω_k	c	$\bar{\tau}$	$\bar{u}$	$\bar{e}$	$\overline{v/u}$
From Table 1	0.19	0.49	0.22	76%	6.1%	0.93	0.49
$\gamma \in [0,1]$	0.19	0.49	0.22	76%	6.1%	0.93	0.49
$\alpha = 1$	0.19	0.49	0.32	56%	4.9%	1.21	0.57
$\eta = 1$	0.15	0.49	0.22	59%	5.0%	1.18	0.40
$\sigma = 0.5$	0.19	0.74	0.22	76%	6.2%	0.92	0.49
$\sigma = 2$	0.19	0.69	0.22	80%	6.6%	0.81	0.52
$\kappa = 0.9$	0.19	0.65	0.22	76%	6.2%	0.90	0.49
$\kappa = 3.6$	0.19	0.41	0.22	78%	6.1%	0.94	0.49
$\alpha = 0.5$	0.19	0.49	0.16	79%	6.2%	0.90	0.50
$\alpha = 0.84$	0.19	0.49	0.27	68%	5.7%	1.05	0.47

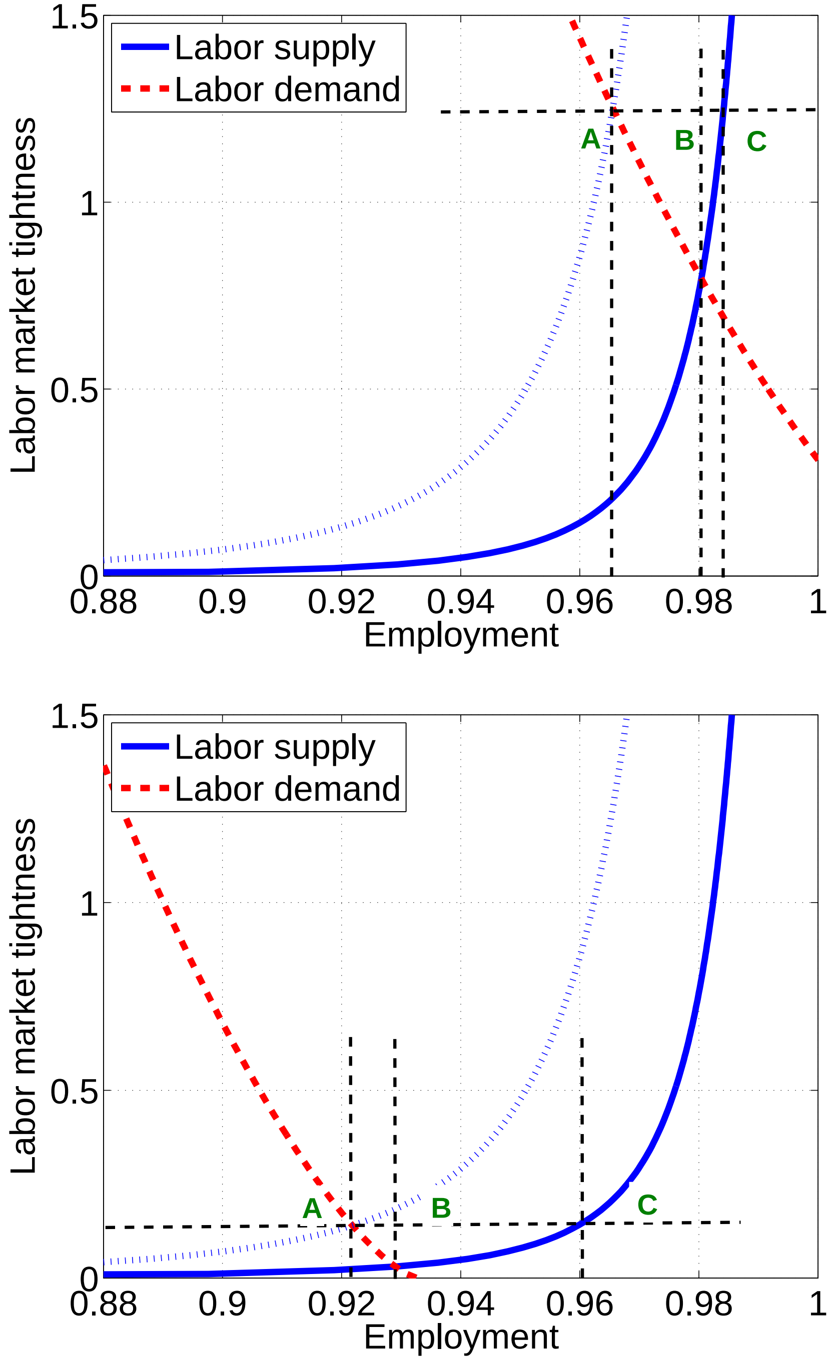

Figure 1: LABOR DEMAND AND LABOR SUPPLY IN ONE-PERIOD MODEL

Notes: These diagrams describe equilibria in the one-period model with job rationing. The two panels represent labor supply and labor demand for high technology $a = 1.03$ (top) and low technology $a = 0.97$ (bottom). The two labor supply curves correspond to a net replacement rate $\tau = 72\%$ calibrated in US data (dotted line) and to a low replacement rate $\tau = 50\%$ (plain line). Diagrams are obtained by plotting labor demand (7) and labor supply, which is a combination of (3) and (5), for $\theta \in [0, 1.5]$. The one-period model is calibrated in Table 1. Note that since we use log-utility, keeping a constant τ imposes a constant $\Delta u = -\ln(\tau)$.

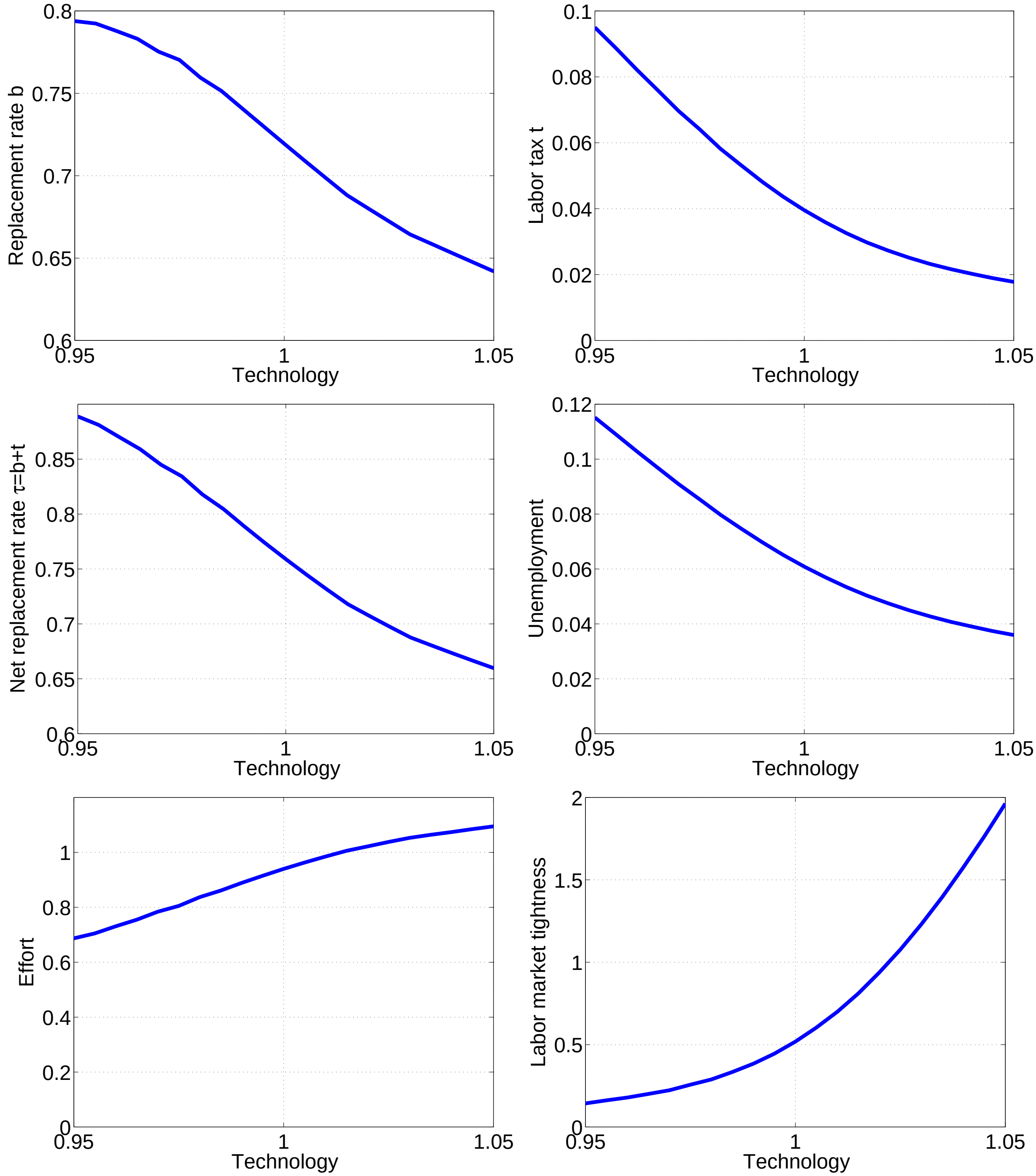

Figure 2: OPTIMAL UI PROGRAM AND EQUILIBRIUM OUTCOMES IN ONE-PERIOD MODEL AS A FUNCTION OF TECHNOLOGY

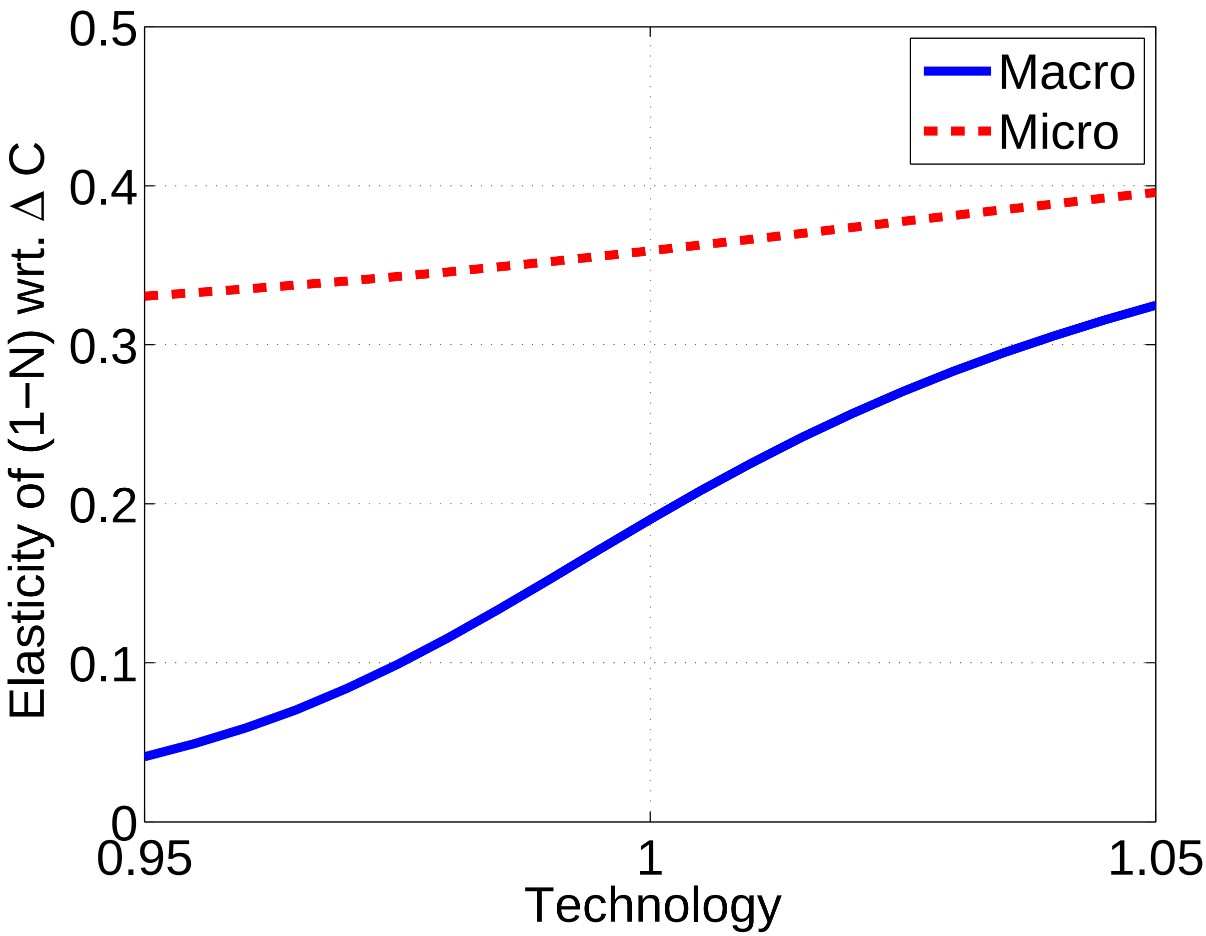

Figure 3: MICRO- AND MACRO-ELASTICITY OF UNEMPLOYMENT WITH RESPECT TO NET REWARD FROM WORK.

Notes: The elasticity of $1-N$ with respect to ΔC is represented as a function of technology. We fix $\tau = 76\%$—equivalent to fixing Δu with log-utility— which is the optimal net replacement rate with steady-state technology $a = 1$.

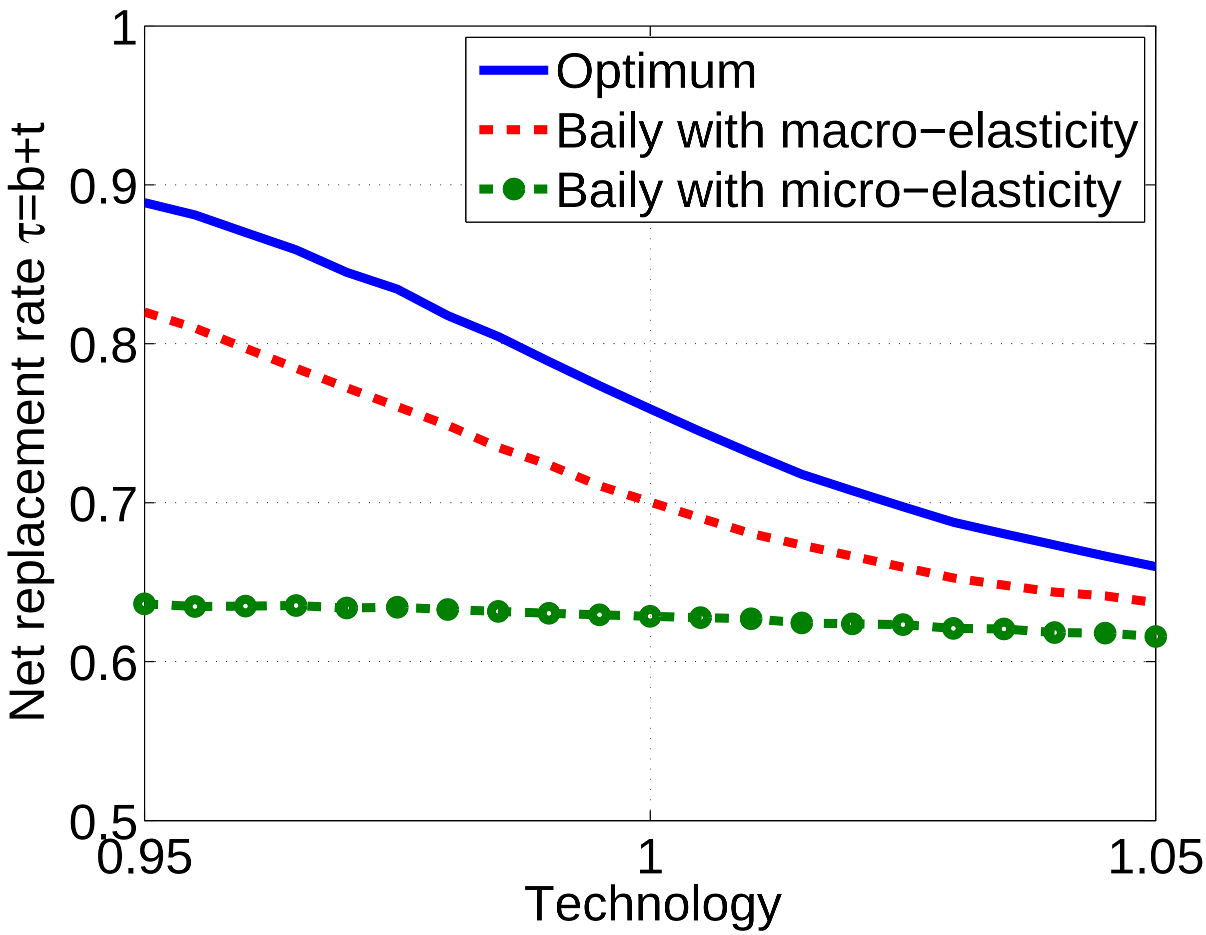

Figure 4: NET REPLACEMENT RATE DERIVED WITH ALTERNATIVE FORMULAS

Notes: The net replacement rate τ is obtained in the one-period model, and represented as a function of technology a. The green (dashed with circles) line is obtained with the Baily formula using the micro-elasticity ε^m of unemployment with respect to net rewards from work. The red (dashed) line is obtained with the Baily formula using the macro-elasticity ε^M of unemployment with respect to net rewards from work. The blue (solid) line is obtained with our optimal formula (12).

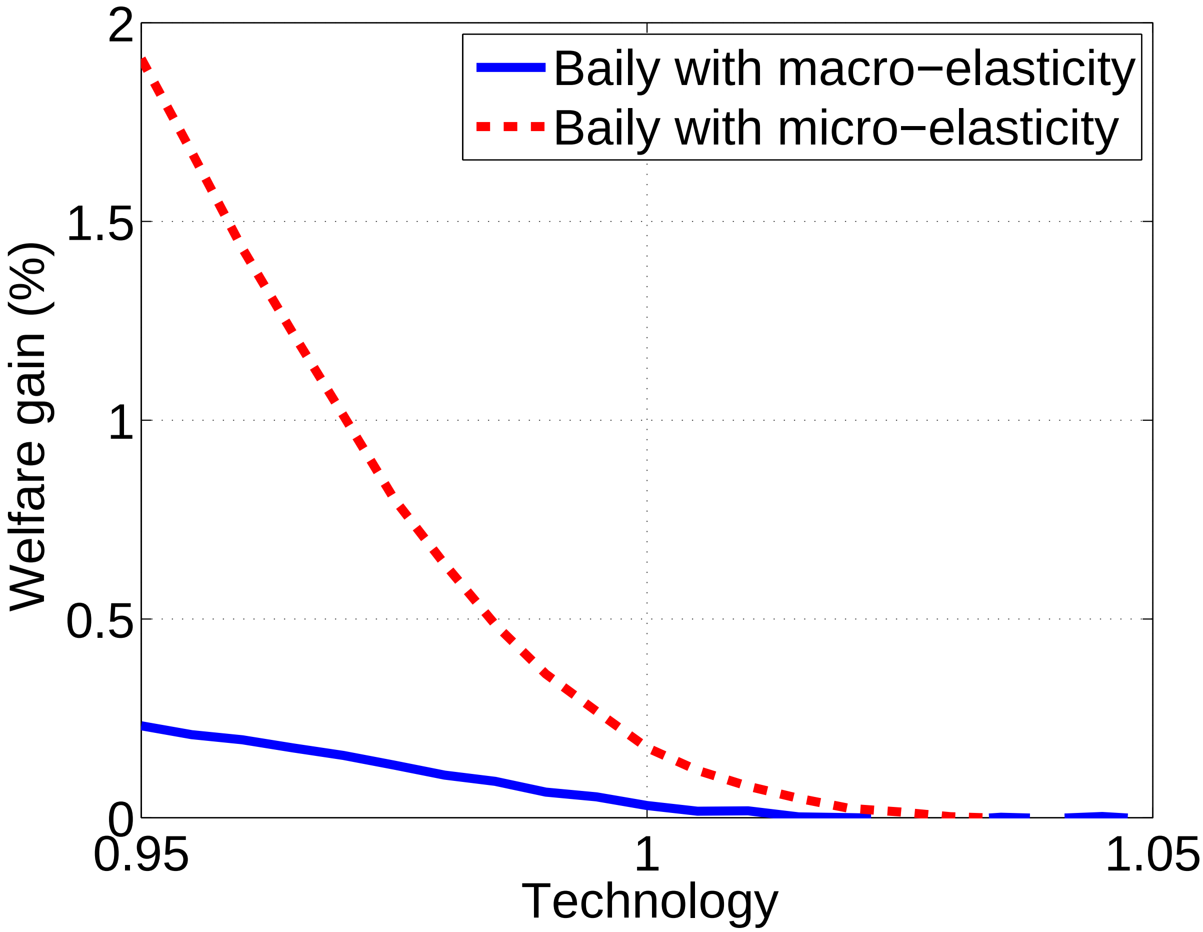

Figure 5: WELFARE GAINS FROM ADOPTING AN OPTIMAL UNEMPLOYMENT INSURANCE

Notes: These welfare gains are percentage increase in certainty-equivalent consumption—the amount of consumption C^{eq} such that $U(C^{eq}) = SW$—from adopting the optimal level of unemployment benefits and labor tax. The welfare gains are measured in the one-period model, as a function of technology. The optimal UI is compared to UI obtained with various Baily formulas.

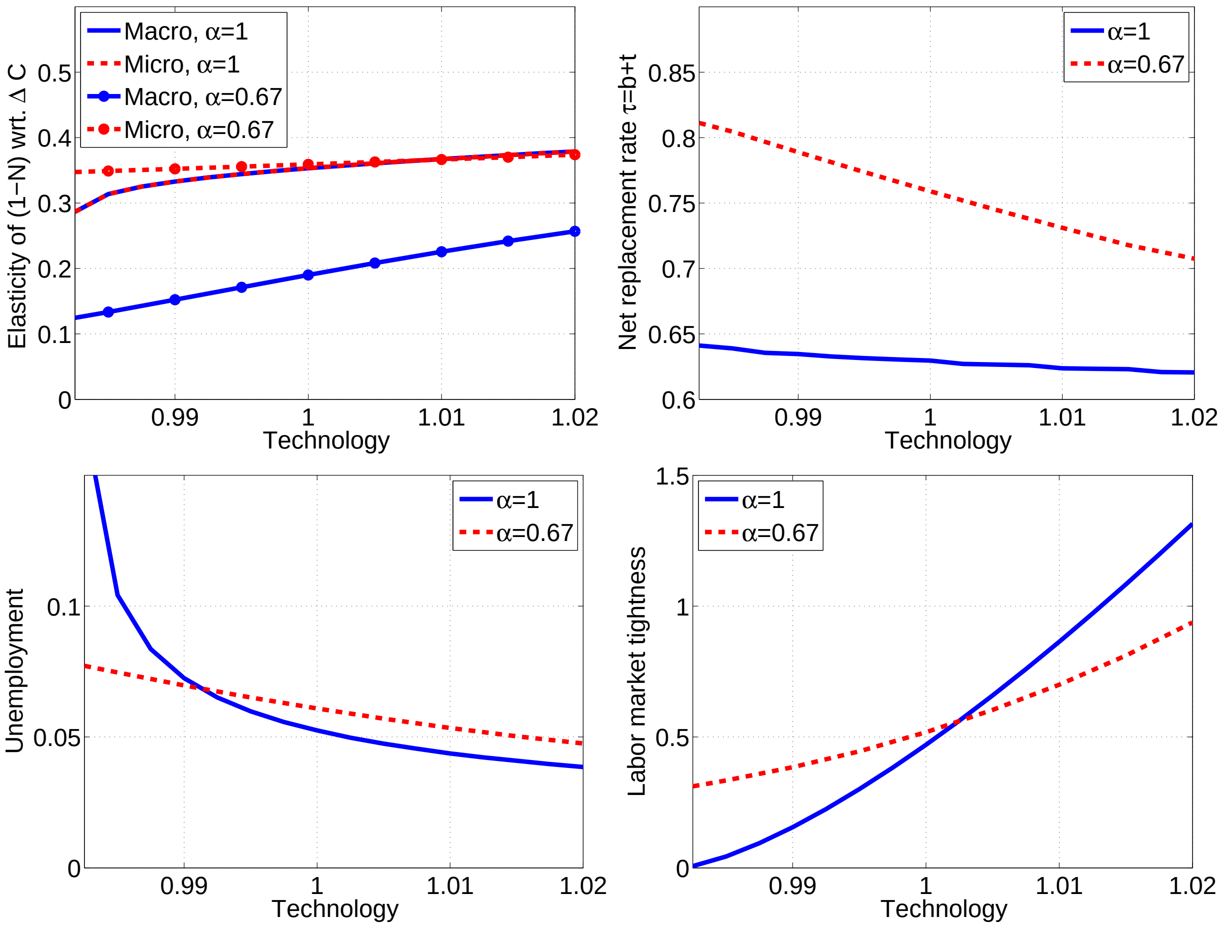

Figure 6: COMPARISON OF ONE-PERIOD MODEL WITH JOB RATIONING ($\alpha = 0.67$) TO MODEL WITHOUT JOB RATIONING ($\alpha = 1$)

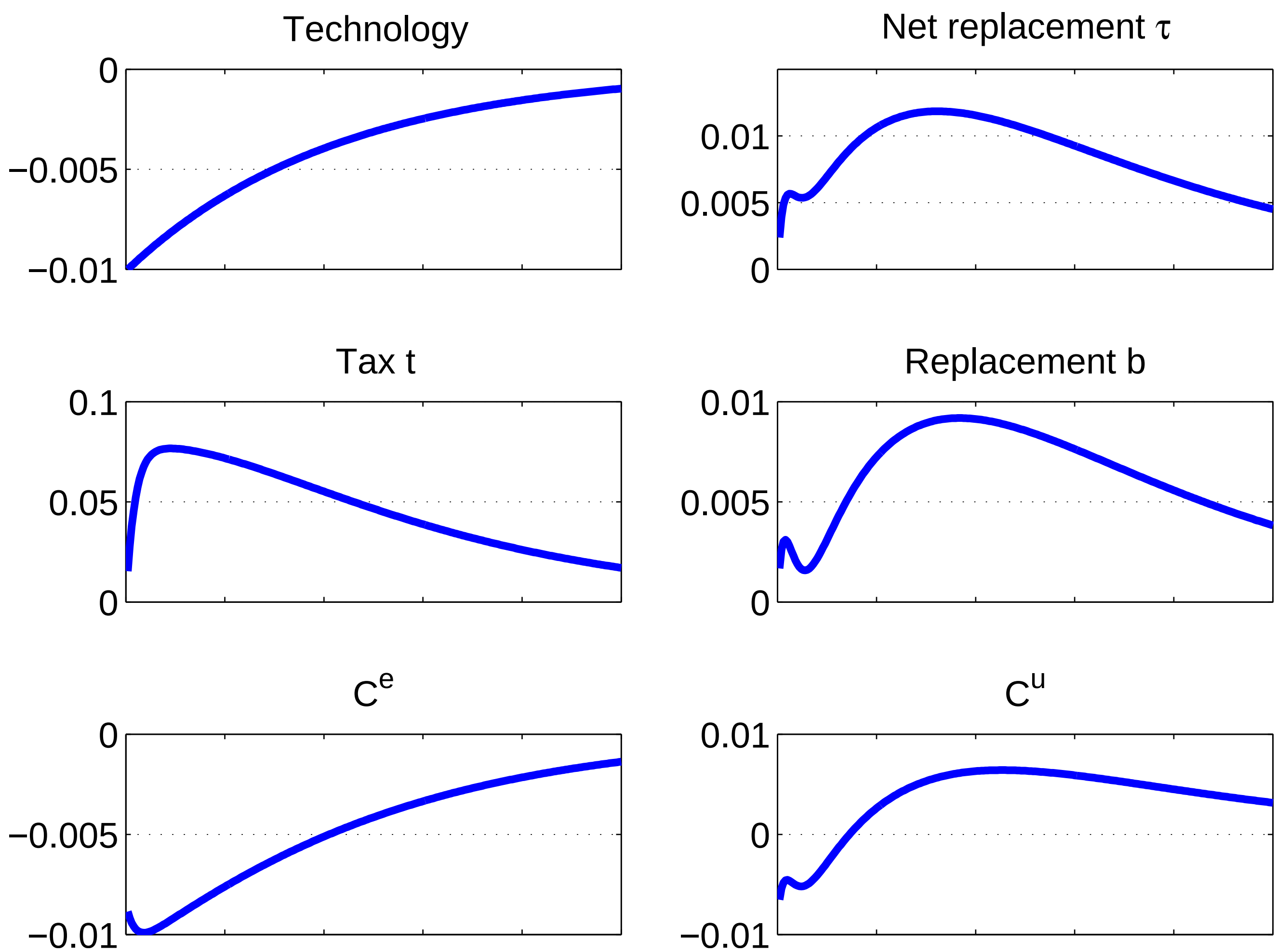

Figure 7: DETAIL OF RESPONSE OF OPTIMAL UI PROGRAM TO A NEGATIVE TECHNOLOGY SHOCK

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$(about 4 times the standard deviation 0.0026) to the log-linear equilibrium describing the Ramsey allocation (allocation with optimal UI). The time period displayed on the x-axis is 250 weeks.

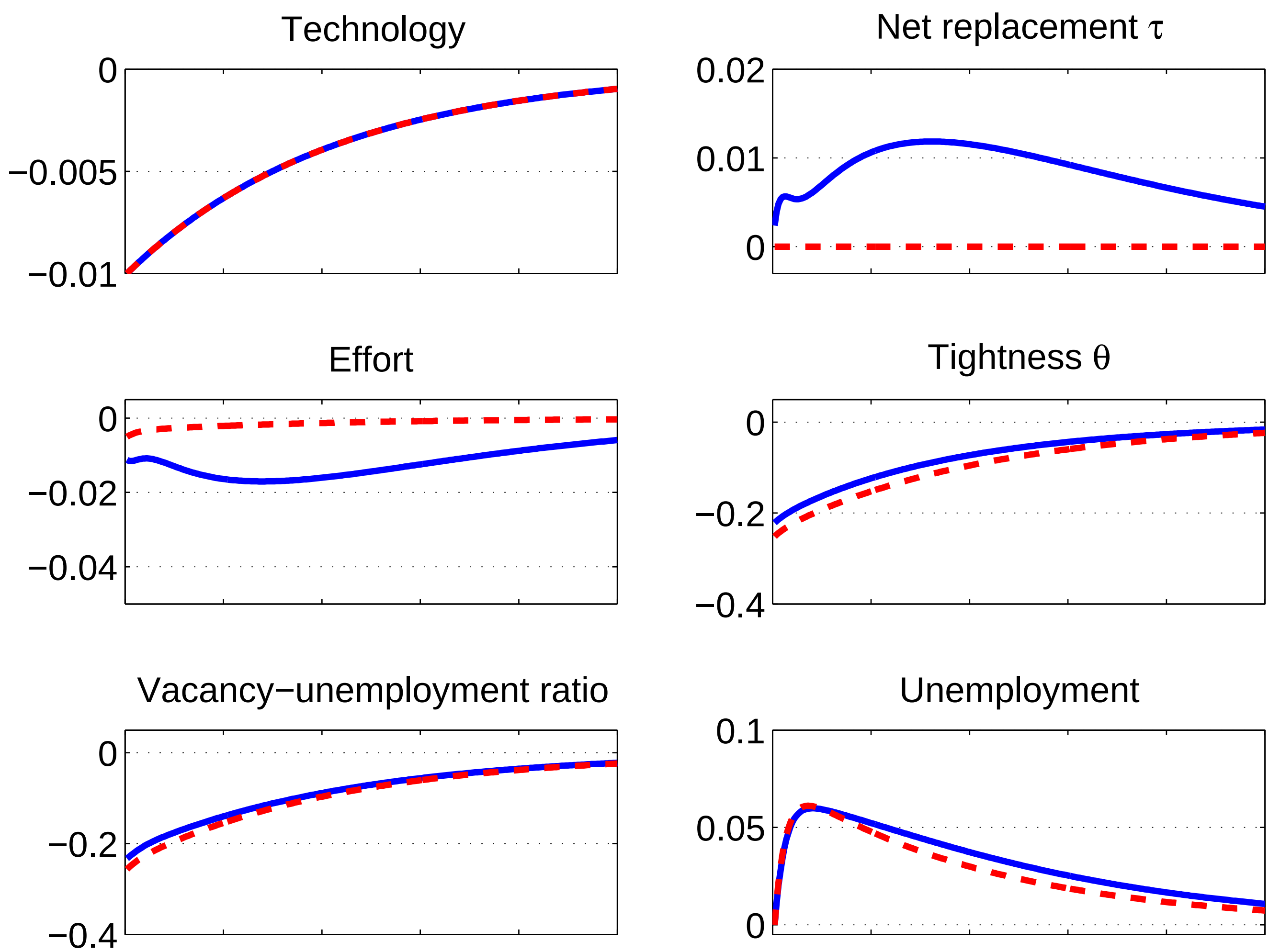

Figure 8: RESPONSE OF OPTIMAL UI PROGRAM AND EQUILIBRIUM OUTCOMES TO A NEGATIVE TECHNOLOGY SHOCK

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue (solid) line IRFs are responses of the Ramsey allocation (allocation with optimal UI). The red (dashed) line IRFs are a useful benchmark: the responses of the economy when the net replacement rate is constant with $\tau = 72\%$.

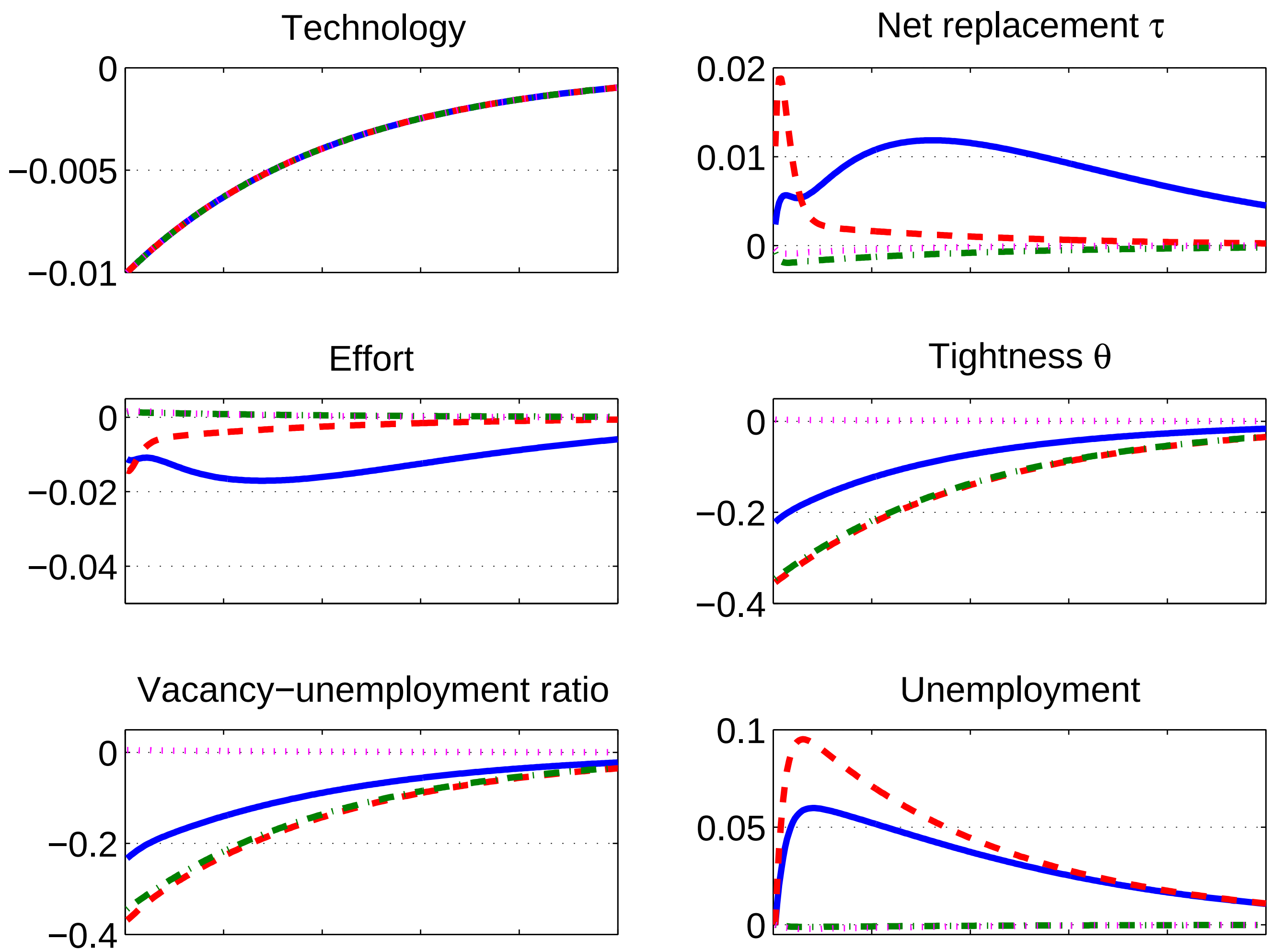

Figure 9: COMPARISON OF RESPONSES OF OPTIMAL UI PROGRAM TO A NEGATIVE TECHNOLOGY SHOCK ACROSS MODELS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue (solid) IRFs are in our base model ($\alpha = 0.67, \eta = 0.7, \gamma = 0.5$). The red (dashed) IRFs are in a model with $\alpha = 1$ (no diminishing returns to labor). The green (dot-dashed) IRFs are in a model with $\eta = 1$. The magenta (dotted) IRFs are in a model with $\gamma = 1$ (no wage rigidities).

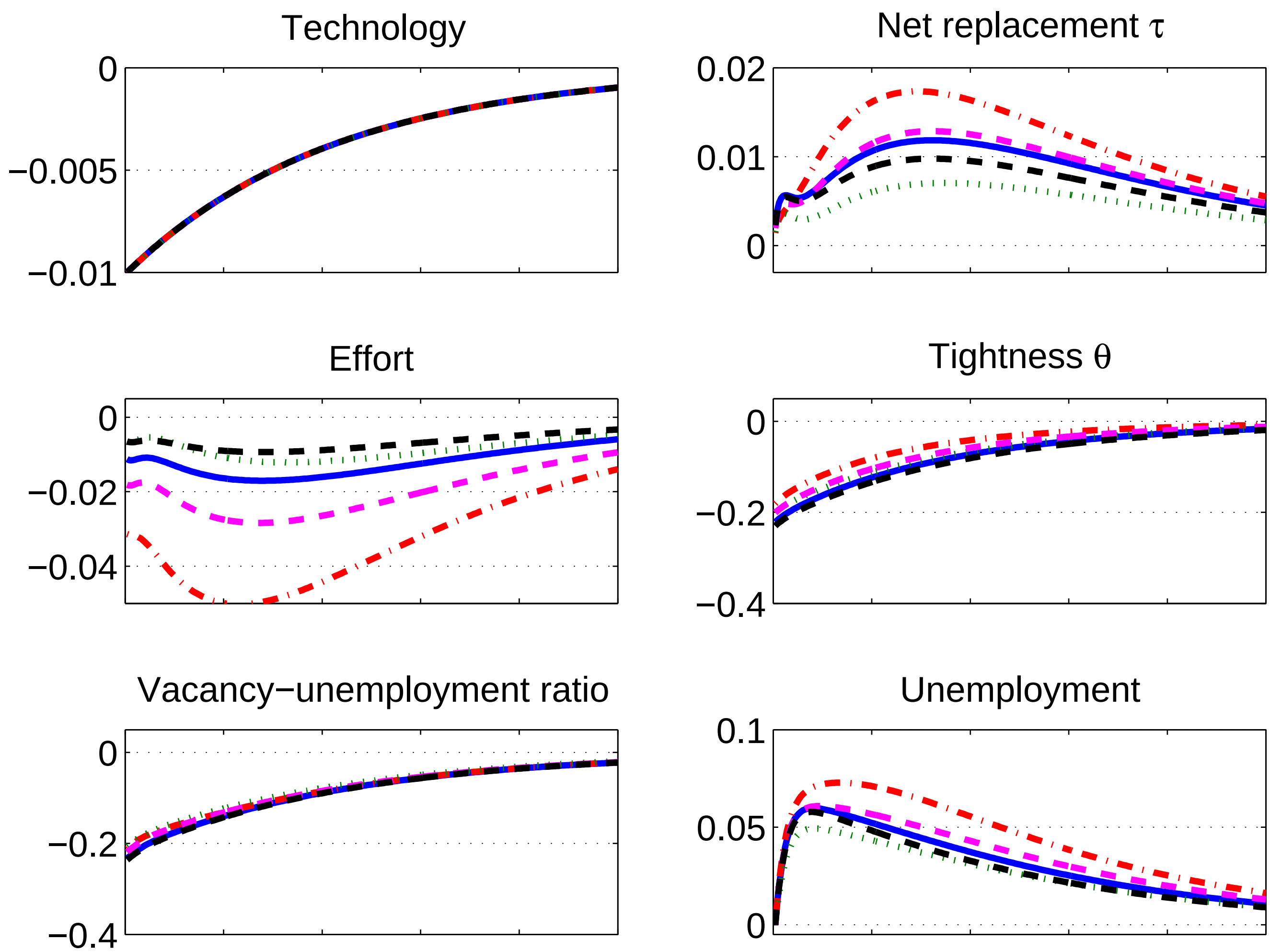

Figure 10: RESPONSES OF OPTIMAL UI TO A NEGATIVE TECHNOLOGY SHOCK FOR ALTERNATIVE UTILITY CALIBRATIONS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue IRFs are in our base model ($\sigma = 1, \kappa = 1.8$). The red (dot-dashed) IRFs are in a model with $\sigma = 0.5$ (less risk aversion). The green (dotted) IRFs are in a model with $\sigma = 2$ (more risk aversion). The magenta (dashed) IRFs are in a model with $\kappa = 0.9$ (larger micro-elasticity). The black (dashed) are in a model with $\kappa = 3.6$ (smaller micro-elasticity).

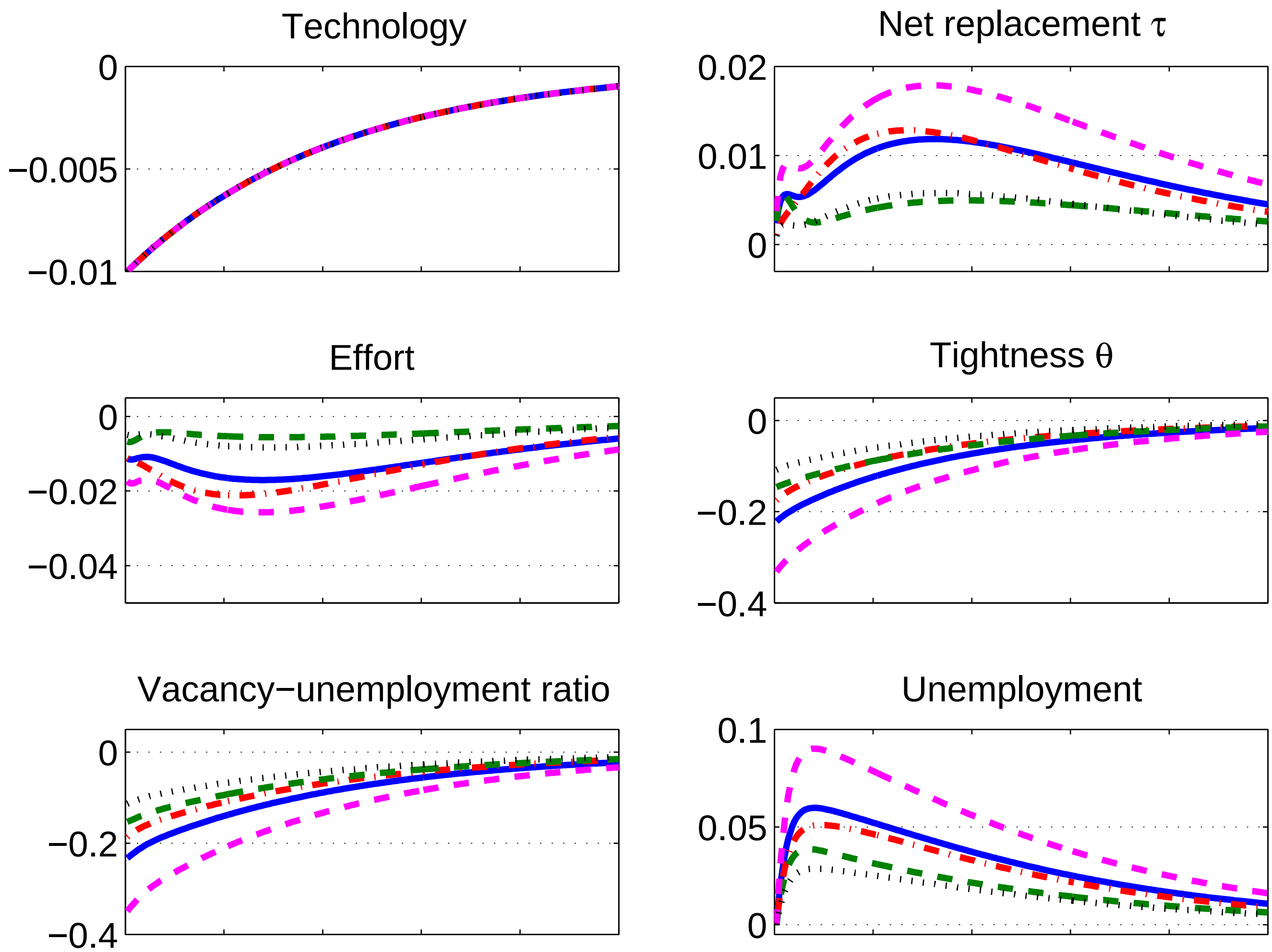

Figure 11: RESPONSES OF OPTIMAL UI TO A NEGATIVE TECHNOLOGY SHOCK FOR ALTERNATIVE JOB-RATIONING CALIBRATIONS

Notes: This figure displays impulse response functions (IRFs), which represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock $z_1 = -0.01$ to the log-linear model (about 4 times the standard deviation 0.0026). The time period displayed on the x-axis is 250 weeks. The blue IRFs are in our base model ($\sigma = 1, \kappa = 1.8$). The red (dot-dashed) IRFs are in a model with $\alpha = 0.5$ (more diminishing returns to labor). The green (dashed) IRFs are in a model with $\alpha = 0.84$ (less diminishing returns to labor). The magenta (dashed) IRFs are in a model with $\gamma = 0.25$ (more wage rigidity). The black (dotted) are in a model with $\gamma = 0.75$ (less wage rigidity).